

Mathematics 101 Midterm 2 Review Package – **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 21 pages, including 1 cover page and 23 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

There is a short table of integrals that ought to be memorized on the last page. If you have not yet committed them to memory, please refer there.

Solutions posted at: <http://ubcengineers.ca/services/academic/tutoring/>

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.



Good Luck!

Integrals You Should Memorize

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \log |x|$$

$$\int e^x dx = e^x$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x dx = \sin x$$

$$\int \sec^2 x dx = \tan x$$

$$\int \sec x \tan x dx = \sec x$$

$$\int \frac{1}{1+x^2} dx = \arctan x$$

- (*) 1. Compute the value of the integral.

$$\int_4^8 \frac{x}{\sqrt{x^2 - 15}} dx$$

- (*) 2. If $x = 6 \cos \theta$, $y = 2 \sin \theta$, compute the value of the integral.

$$\int_3^6 xy dx$$

(**) 3. Evaluate the integral.

$$\int \frac{e^{2x}}{1 + e^{4x}} dx$$

(**) 4. Evaluate the integral.

$$\int \frac{2x - 7}{x^2 + 1} dx$$

(**) 5. Compute the value of the integral.

$$\int_4^9 \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx$$

(**) 6. Evaluate the integral.

$$\int \frac{e^x - 1}{e^x + 1} dx$$

(**) 7. Evaluate the integral

$$\int_0^{\pi} e^{\cos t} \sin(2t) dt$$

(**) 8. Evaluate the integral

$$\int \cos(\sqrt{x}) dx$$

(**) 9. Evaluate the integral.

$$\int \frac{1}{1 - \sin(x/2)} dx$$

(***) 10. Evaluate the integral.

$$\int \frac{\cos 2x}{\sin^2(2x) + 8} dx$$

(***) 11. Compute the value of the integral.

$$\int_0^{\pi} \frac{\sin x}{\cos^2 x - 6 \cos x + 13} dx$$

(***) 12. Evaluate the integral

$$\int x \arctan x dx$$

(**) 13. Evaluate the integral

$$\int \cos(\log x) dx$$

(**) 14. Evaluate the integral.

$$\int (\sin x)(\sin 3x)dx$$

(***) 15. Evaluate the integral.

$$\int \sqrt{1 - \cos x} dx$$

- (*) 16. Find the volume of the solid generated by revolving the plane area bounded by $x - y - 7 = 0$, $x = 9 - y^2$ about the y -axis.

- (*) 17. Find the volume of the solid generated by revolving the plane area bounded by $y = x^3$, $y = 0$, $x = 2$, about the line $x = 2$.

- (*) 18. Find the volume of the solid generated by revolving the plane area bounded by $y = x^2$ and $y = 4x - x^2$ around the line $y = 6$.

- (**) 19. Compute the area enclosed by the curves: $y = 25 - x^2$, $256x = 3y^2$, $16y = 9x^2$

(***) 20. Compute the area enclosed by the curve $y^2 = x^2 - x^4$.

Hint. *This curve is symmetric with respect to the x axis and the y axis. How can you use symmetry to help you calculate the area?*

- (***) 21. Compute the area enclosed by the curve $y^2 = x^4(x + 4)$ for the portion of the graph lying to the left of the y -axis.

- (***) 22. A solid has a base in the form of an ellipse with major diameter 10 and minor diameter 8. Find the volume if every section perpendicular to the major axis is an isosceles triangle with altitude 6.

- (***) 23. The base of a solid is the circle $x^2 + y^2 = 16x$, and every plane section perpendicular to the x -axis is a rectangle whose height is twice the distance of the plane of the section from the origin. Find the volume of this solid.