Mathematics 101 Quiz 3 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 19 pages, including 1 cover page and 20 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(***)$ for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a $(*)$ problem.

Solutions posted at: `http://ubcengineers.ca/services/academic/tutoring/`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!

Integrals You Should Definitely Memorize

$$\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1}, \quad n \neq -1 \\
\int \frac{1}{x} dx &= \log|x| \\
\int e^x dx &= e^x \\
\int \sin x dx &= -\cos x \\
\int \cos x dx &= \sin x \\
\int \sec^2 x dx &= \tan x \\
\int \sec x \tan x dx &= \sec x \\
\int \frac{1}{1+x^2} dx &= \arctan x
\end{aligned}$$

Integrals You Might Want to Memorize But Are Less Important

$$\begin{aligned}
\int \sec x dx &= \log|\tan x + \sec x| \\
\int \csc x dx &= -\log|\csc x + \cot x| \\
\int \log x dx &= x \log x - x \\
\int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin x \\
\int \frac{-1}{\sqrt{1-x^2}} dx &= \arccos x
\end{aligned}$$

(∗) 1. Evaluate the integral

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$$

(∗∗) 2. Evaluate the integral.

$$\int \sin^3 x \cos^2 x dx$$

(∗∗) 3. Evaluate the integral.

$$\int \tan^4 x \sec^4 x dx$$

(∗) 4. Evaluate the integral.

$$\int \frac{dx}{x^2 + 7x + 6}$$

(∗∗) 5. Evaluate the integral.

$$\int \sin^4 x dx$$

(∗∗) 6. Evaluate the integral.

$$\int x(\cos^3(x^2) - \sin^3(x^2))dx$$

(∗∗) 7. Evaluate the integral.

$$\int \frac{dx}{(9+x^2)^2}$$

(∗∗) 8. Evaluate the integral.

$$\int \frac{x+1}{x^3+x^2-6x}dx$$

(∗∗) 9. Evaluate the integral

$$\int x\sqrt{1-x^4}dx$$

(∗∗) 10. Evaluate the integral.

$$\int \frac{(16-9x^2)^{3/2}}{x^6}dx$$

(∗∗) 11. Evaluate the integral

$$\int \frac{e^x}{(e^x-2)(e^{2x}+1)}dx$$

(∗∗) 12. Determine how large n must be in order to guarantee that the trapezoidal estimate for the integral $\int_1^2 \frac{1}{x} dx$ differs from its true value by no more than 0.0005, where the error is given by $E_T = \dfrac{K(b-a)^3}{12n^2}$, $K \geq |f''(x)|$, $x \in [a, b]$.

(∗∗) 13. Approximate the area under $f(x) = \dfrac{1}{x^2+1}$ for $x \in [0, 1/2]$ by

(a) Using the trapezoidal rule, $n = 5$

(b) Using Simpson's rule, $n = 4$

You may leave your answers in calculator–ready form.

(∗∗) 14. A function f is given by $f(x) = \int_1^x \sqrt{1+\sin t}\,dt$. Use Simpson's rule with 6 subintervals to approximate $f(3)$.You may leave your answer in calculator–ready form.

(∗∗) 15. Evaluate the integral. The attached table may be useful.

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin^2\theta}}\,d\theta$$

(∗∗) 16. Evaluate the integral

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx$$

(∗ ∗ ∗) 17. Evaluate the integral.

$$\int \frac{2x-3}{x^2+6x+13}dx$$

(∗ ∗ ∗) 18. Evaluate the integral without referring to the attached table.

$$\int \sec x dx$$

(∗ ∗ ∗) 19. Evaluate the integral. The attached table may be useful.

$$\int \frac{dx}{x\sqrt{9+4x^2}}$$

(∗ ∗ ∗) 20. Evaluate the integral. The attached table may be useful.

$$\int \frac{x^2}{\sqrt{x^2-16}}dx$$