Mathematics 101 Quiz 3 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 14 pages, including 1 cover page and 20 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(***)$ for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a $(*)$ problem.

Solutions posted at: `http://ubcengineers.ca/services/academic/tutoring/`

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Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!

Integrals You Should Definitely Memorize

$$
\begin{aligned}
\int x^n dx &= \frac{x^{n+1}}{n+1}, \quad n \neq -1 \\
\int \frac{1}{x} dx &= \log|x| \\
\int e^x dx &= e^x \\
\int \sin x dx &= -\cos x \\
\int \cos x dx &= \sin x \\
\int \sec^2 x dx &= \tan x \\
\int \sec x \tan x dx &= \sec x \\
\int \frac{1}{1+x^2} dx &= \arctan x
\end{aligned}
$$

Integrals You Might Want to Memorize But Are Less Important

$$
\begin{aligned}
\int \sec x dx &= \log|\tan x + \sec x| \\
\int \csc x dx &= -\log|\csc x + \cot x| \\
\int \log x dx &= x \log x - x \\
\int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin x \\
\int \frac{-1}{\sqrt{1-x^2}} dx &= \arccos x
\end{aligned}
$$

(∗) 1. Evaluate the integral

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$$

Solution: Let $u = 1 + \sqrt{x}$, and $2du = \frac{1}{\sqrt{x}}dx$. Then the integral becomes

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = 2\int \sqrt{u}du$$

This integral is easily evaluated

$$2\int \sqrt{u}du = \frac{4}{3}u^{3/2} + C$$

Now changing to the original variables,

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \frac{4}{3}(1+\sqrt{x})^{3/2} + C$$

(∗∗) 2. Evaluate the integral.

$$\int \sin^3 x \cos^2 x dx$$

Solution: Let $\cos x = u$, $du = -\sin x dx$

$$\begin{aligned}\int \sin^3 x \cos^2 x dx &= -\int (1-u^2)u^2 du \\ &= \int u^4 - u^2 du \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C \\ &= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C\end{aligned}$$

$$\int \sin^3 x \cos^2 x dx = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$$

(∗∗) 3. Evaluate the integral.

$$\int \tan^4 x \sec^4 x dx$$

Solution: Let $u = \tan x$, $du = \sec^2 x dx$

$$\begin{aligned} \int u^4(u^2+1)du &= \int u^6 + u^4 du \\ &= \frac{u^7}{7} + \frac{u^5}{5} + C \\ &= \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C \end{aligned}$$

$$\int \tan^4 x \sec^4 x dx = \frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$$

(∗) 4. Evaluate the integral.

$$\int \frac{dx}{x^2 + 7x + 6}$$

Solution:

$$\begin{aligned} \int \frac{dx}{x^2+7x+6} &= \int \frac{dx}{(x+6)(x+1)} \\ &= \frac{1}{5}\int \frac{1}{x+1} - \frac{1}{x+6} dx \\ &= \frac{1}{5}\log\left|\frac{x+1}{x+6}\right| + C \end{aligned}$$

(∗∗) 5. Evaluate the integral.

$$\int \sin^4 x dx$$

Solution:

$$\begin{aligned}
\int \sin^4 x dx &= \int \sin^2 x(1-\cos^2 x)dx \\
&= \int \sin^2 x - \sin^2 x \cos^2 x dx \\
&= \int \sin^2 x dx - \int \sin^2 x \cos^2 x dx \\
&= \int \frac{1-\cos 2x}{2} dx - \int \frac{\sin^2 2x}{4} dx \\
&= \frac{1}{2}x - \frac{\sin 2x}{4} - \int \frac{1-\cos 4x}{8} dx \\
&= \frac{1}{2}x - \frac{\sin 2x}{4} - \frac{1}{8}x + \frac{\sin 4x}{32} + C \\
&= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C
\end{aligned}$$

$$\int \sin^4 x dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

(**) 6. Evaluate the integral.

$$\int x(\cos^3(x^2) - \sin^3(x^2))dx$$

Solution: Let $x^2 = u$, $du = 2xdx$. Then

$$\begin{aligned}
\int x(\cos^3(x^2) - \sin^3(x^2))dx &= \frac{1}{2}\int \cos^3 u - \sin^3 u du \\
&= \frac{1}{2}\int \cos^3 u du - \frac{1}{2}\int \sin^3 u du \\
&= \frac{1}{2}\int (1-\sin^2 u)\cos u du - \frac{1}{2}\int (1-\cos^2 u)\sin u du \\
&= \frac{1}{2}\int \cos u - \sin^2 u \cos u du - \frac{1}{2}\int \sin u - \cos^2 u \sin u du \\
&= \frac{1}{2}\sin u - \frac{1}{6}\sin^3 u + \frac{1}{2}\cos u - \frac{1}{6}\cos^3 u \\
&= \frac{1}{12}\left(6(\sin u + \cos u) - 2(\sin^3 u + \cos^3 u)\right) \\
&= \frac{1}{12}\left(6(\sin u + \cos u) - 2(\sin u + \cos u)(\sin^2 u + \cos^2 u - \sin u \cos u)\right) \\
&= \frac{\sin u + \cos u}{12}(6 - (2 - \sin 2u)) + C \\
&= \frac{1}{12}(\sin(x^2) + \cos(x^2))(4 + \sin(2x^2)) + C
\end{aligned}$$

Thus the final answer is

$$\int x(\cos^3(x^2) - \sin^3(x^2))dx = \frac{1}{12}(\sin(x^2) + \cos(x^2))(4 + \sin(2x^2)) + C$$

(∗∗) 7. Evaluate the integral.

$$\int \frac{dx}{(9 + x^2)^2}$$

Solution: We recognize this as the form of a trigonometric substitution integral, so let $x = 3\tan z$, and $dx = 3\sec^2 z dz$.

$$\begin{aligned}
\int \frac{3\sec^2 z dz}{(9 + 9\tan^2 z)^2} &= \frac{1}{27}\int \frac{1}{\sec^2 z}dz \\
&= \frac{1}{27}\int \cos^2 z dz \\
&= \frac{1}{54}\int 1 + \cos 2z dz \\
&= \frac{1}{54}\left(z + \frac{\sin 2z}{2}\right) \\
&= \frac{1}{54}\left(\arctan\left(\frac{x}{3}\right) + \sin z \cos z\right) \\
&= \frac{1}{54}\left(\arctan\left(\frac{x}{3}\right) + \sin z \cos z\right) \\
&= \frac{1}{54}\left(\arctan\left(\frac{x}{3}\right) + \frac{3x}{x^2 + 9}\right) + C
\end{aligned}$$

The final answer is then

$$\int \frac{dx}{(9 + x^2)^2} = \frac{1}{54}\arctan\left(\frac{x}{3}\right) + \frac{x}{18(x^2 + 9)} + C$$

(∗∗) 8. Evaluate the integral.

$$\int \frac{x + 1}{x^3 + x^2 - 6x}dx$$

Solution:

$$\begin{aligned}
\int \frac{x + 1}{x^3 + x^2 - 6x}dx &= \int \frac{x + 1}{x(x + 3)(x - 2)}dx \\
&= \int -\frac{1}{6x} + \frac{3}{10(x - 2)} - \frac{2}{15(x + 3)}dx \\
&= -\frac{1}{6}\log|x| + \frac{3}{10}\log|x - 2| - \frac{2}{15}\log|x + 3| + C
\end{aligned}$$

(**) 9. Evaluate the integral

$$\int x\sqrt{1-x^4}dx$$

Solution: Let $x^2 = \sin\theta$, and $2xdx = \cos\theta d\theta$. Then the integral becomes

$$\begin{aligned}\int x\sqrt{1-x^4}dx &= \frac{1}{2}\int \sqrt{1-\sin^2\theta}\cos\theta d\theta \\ &= \frac{1}{2}\int \cos^2\theta d\theta \\ &= \frac{1}{2}\int \frac{1+\cos 2\theta}{2}d\theta \\ &= \frac{\theta}{4} + \frac{\sin 2\theta}{8} + C \\ &= \frac{\theta}{4} + \frac{\sin\theta\cos\theta}{4} + C\end{aligned}$$

Now plugging back in the original variables we have

$$\int x\sqrt{1-x^4}dx = \frac{\arcsin(x^2)}{4} + \frac{x^2\sqrt{1-x^4}}{4} + C$$

(**) 10. Evaluate the integral.

$$\int \frac{(16-9x^2)^{3/2}}{x^6}dx$$

Solution: Let $x = \frac{4}{3}\sin\theta$, so $dx = \frac{4}{3}\cos\theta d\theta$. Thus the integral becomes

$$\begin{aligned}\int \frac{(16-9x^2)^{3/2}}{x^6}dx &= \int \frac{64\cos^3\theta}{\frac{4^6}{3^6}\sin^6\theta}\frac{4}{3}\cos\theta d\theta \\ &= \frac{3^5}{2^4}\int \cot^4\theta\csc^2\theta d\theta\end{aligned}$$

Now we make a second substitution. Let $\cot\theta = u$, so $du = -\csc^2\theta d\theta$. Then we have the greatly simplified integral

$$\begin{aligned}\frac{3^5}{2^4}\int \cot^4\theta\csc^2\theta d\theta &= -\frac{3^5}{2^4}\int u^4 du \\ &= -\frac{3^5}{2^4}\frac{u^5}{5} + C \\ &= -\frac{3^5}{2^4}\frac{\cot^5\theta}{5} + C \\ &= -\frac{3^5}{2^4}\frac{1}{5}\left(\frac{\sqrt{16-9x^2}}{3x}\right)^5 + C \\ &= -\frac{1}{80}\frac{(16-9x^2)^{5/2}}{x^5} + C\end{aligned}$$

This leaves us with the final answer

$$\int \frac{(16-9x^2)^{3/2}}{x^6}dx = -\frac{1}{80}\frac{(16-9x^2)^{5/2}}{x^5} + C$$

(**) 11. Evaluate the integral

$$\int \frac{e^x}{(e^x-2)(e^{2x}+1)}dx$$

Solution: Let $e^x = u$. Then $du = e^x dx$. The integral is then transformed to

$$\int \frac{e^x}{(e^x-2)(e^{2x}+1)}dx = \int \frac{du}{(u-2)(u^2+1)}$$

We apply a partial fraction decomposition to this. Just looking at the integrand,

$$\frac{1}{(u-2)(u^2+1)} = \frac{1}{5(x-2)} - \frac{x}{5(x^2+1)} - \frac{2}{5(x^2+1)}$$

Now integrating term by term, we have

$$\int \frac{du}{(u-2)(u^2+1)} = \frac{1}{5}\log|u-2| - \frac{1}{10}\log(u^2+1) - \frac{2}{5}\arctan u + C$$

Now transforming back to the original variable x,

$$\int \frac{e^x}{(e^x-2)(e^{2x}+1)}dx = \frac{1}{5}\log|e^x-2| - \frac{1}{10}\log(e^{2x}+1) - \frac{2}{5}\arctan e^x + C$$

(**) 12. Determine how large n must be in order to guarantee that the trapezoidal estimate for the integral $\int_1^2 \frac{1}{x}dx$ differs from its true value by no more than 0.0005, where the error is given by $E_T = \frac{K(b-a)^3}{12n^2}$, $K \geq |f''(x)|$, $x \in [a,b]$.

Solution: We require the error to satisfy

$$E_T = \frac{K(b-a)^3}{12n^2} \leq 0.0005 = 5 \cdot 10^{-4}$$

We know that the second derivative is given by $f''(x) = \frac{2}{x^3}$, the maximum of which on $[1,2]$ is $K = 2$. Thus the error term becomes

$$E_T = \frac{2}{12n^2} \leq 5 \cdot 10^{-4}$$

Now solving this inequality for n. Just rearranging

$$1 \geq 3 \cdot 10^{-3} n^2$$

and isolating n,

$$n^2 \geq \frac{1}{3}10^3 \approx 3.33 \cdot 10^2$$

Now approximately evaluating the square root of both sides we obtain

$$n \geq \sqrt{3.33 \cdot 10^2} \approx 20$$

Thus

$$n \geq 20$$

(**) 13. Approximate the area under $f(x) = \dfrac{1}{x^2+1}$ for $x \in [0, 1/2]$ by

(a) Using the trapezoidal rule, $n = 5$

(b) Using Simpson's rule, $n = 4$

You may leave your answers in calculator–ready form.

Solution:

(a)

$$\begin{aligned} T_5 &= \frac{\Delta x}{2}\left(f(0) + 2f\left(\frac{1}{10}\right) + 2f\left(\frac{2}{10}\right) + 2f\left(\frac{3}{10}\right) + 2f\left(\frac{4}{10}\right) + f\left(\frac{5}{10}\right)\right) \\ &= \frac{0.1}{2}\left(1 + \frac{2}{1.01} + \frac{2}{1.04} + \frac{2}{1.09} + \frac{2}{1.16} + \frac{1}{1.25}\right) = 0.4631 \end{aligned}$$

(b)

$$\begin{aligned} S_5 &= \frac{\Delta x}{3}\left(f(0) + 4f\left(\frac{1}{8}\right) + 2f\left(\frac{2}{8}\right) + 4f\left(\frac{3}{8}\right) + f\left(\frac{4}{8}\right)\right) \\ &= \frac{0.125}{3}\left(1 + \frac{256}{65} + \frac{32}{17} + \frac{256}{73} + \frac{4}{5}\right) = 0.4637 \end{aligned}$$

(**) 14. A function f is given by $f(x) = \int_1^x \sqrt{1+\sin t}\,dt$. Use Simpson's rule with 6 subintervals to approximate $f(3)$.You may leave your answer in calculator–ready form.

Solution:

$$f(x) = \int_1^x \sqrt{1+\sin t}\,dt$$

$$\begin{aligned}
f(3) &= \int_1^3 \sqrt{1+\sin t}dt \\
&\approx S_6 \\
&= \frac{\Delta x}{3}\left(f(1)+2f\left(\frac{4}{3}\right)+4f\left(\frac{5}{3}\right)+2f\left(\frac{6}{3}\right)+4f\left(\frac{7}{3}\right)+2f\left(\frac{8}{3}\right)+f(3)\right) \\
&= \frac{1/3}{3}\left(\sqrt{1+\sin 1}+4\sqrt{1+\sin\left(\frac{4}{3}\right)}+2\sqrt{1+\sin\left(\frac{5}{3}\right)}\right. \\
&+ \left. 4\sqrt{1+\sin\left(\frac{6}{3}\right)}+2\sqrt{1+\sin\left(\frac{7}{3}\right)}+4\sqrt{1+\sin\left(\frac{8}{3}\right)}+\sqrt{1+\sin(3)}\right) \\
&= 2.6498
\end{aligned}$$

(∗∗) 15. Evaluate the integral. The attached table may be useful.

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin^2\theta}}d\theta$$

Solution: Let $\sin\theta = u$, and $du = \cos\theta d\theta$. Then we have the integral

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin^2\theta}}d\theta = \int_0^1 \frac{du}{\sqrt{1+u^2}}$$

Let $u = \tan\varphi$, $du = \sec^2\varphi d\varphi$. Then the integral is now

$$\int_0^1 \frac{du}{\sqrt{1+u^2}} = \int_0^{\pi/4} \sec\varphi d\varphi$$

By problem 18, we have

$$\int_0^{\pi/4} \sec\varphi d\varphi = \left(\log|\sec\varphi+\tan\varphi|\right)\Big|_0^{\pi/4}$$

This evaluates to

$$\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1+\sin^2\theta}}d\theta = \log(1+\sqrt{2})$$

(∗∗) 16. Evaluate the integral

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx$$

Solution: Let $x = u^3$. Then $dx = 3u^2du$, and the integral becomes

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx = \int_0^1 \frac{3u^2}{1+u}du$$

Just looking at the integrand, it can be expanded by partial fractions to obtain

$$\frac{3u^2}{1+u} = 3\left(u - 1 + \frac{1}{u+1}\right)$$

Now plugging this back into the integral,

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx = 3\int_0^1 \left(u - 1 + \frac{1}{u+1}\right) du$$

This is easily evaluated as

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx = 3\left(\frac{u^2}{2} - u + \log|u+1|\right)\Big|_0^1$$

Finally we have

$$\int_0^1 \frac{1}{1+\sqrt[3]{x}}dx = 3\log 2 - \frac{3}{2}$$

(∗ ∗ ∗) 17. Evaluate the integral.

$$\int \frac{2x-3}{x^2+6x+13}dx$$

Solution: We will split this integral up so that part of it can be evaluated as a logarithm:

$$\begin{aligned}
\int \frac{2x-3}{x^2+6x+13}dx &= \int \frac{2x+6-9}{x^2+6x+13}dx \\
&= \int \frac{2x+6}{x^2+6x+13}dx - \int \frac{9}{x^2+6x+9+4}dx \\
&= \log\left|x^2+6x+13\right| - 9\int \frac{1}{(x+3)^2+4}dx
\end{aligned}$$

To evaluate the second integral,

$$9\int \frac{1}{(x+3)^2+4}dx$$

we need to make the substitution $x+3 = 2\tan z$, and $dx = 2\sec^2 z dz$. Thus

$$9\int \frac{1}{(x+3)^2+4}dx = \frac{9}{2}\int 1dz = \frac{9z}{2} = \frac{9}{2}\arctan\left(\frac{x+3}{2}\right)$$

The final answer is then

$$\int \frac{2x-3}{x^2+6x+13}dx = \log\left|x^2+6x+13\right| - \frac{9}{2}\arctan\left(\frac{x+3}{2}\right) + C$$

(∗ ∗ ∗) 18. Evaluate the integral without referring to the attached table.

$$\int \sec x dx$$

Solution: We first manipulate the integrand to make it possible to use partial fractions.

$$\begin{aligned}\int \sec\theta d\theta &= \int \frac{1}{\cos\theta}d\theta \\ &= \int \frac{\cos\theta}{\cos^2\theta}d\theta \\ &= \int \frac{\cos\theta}{1-\sin^2\theta}d\theta\end{aligned}$$

Let $\sin\theta = u$, and $du = \cos\theta d\theta$. This transforms the integral in to

$$\begin{aligned}\int \frac{\cos\theta}{1-\sin^2\theta}d\theta &= \int \frac{du}{1-u^2} \\ &= \frac{1}{2}\int \frac{1}{1+u} + \frac{1}{1-u}du \\ &= \frac{1}{2}\log\left|\frac{1+u}{1-u}\right| = \frac{1}{2}\log\left|\frac{1+\sin\theta}{1-\sin\theta}\right| \\ &= \frac{1}{2}\log\left|\frac{1+\sin\theta}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)\right| \\ &= \frac{1}{2}\log\left|\frac{(1+\sin\theta)^2}{\cos^2\theta}\right| \\ &= \log\left|\frac{1+\sin\theta}{\cos\theta}\right| \\ &= \log|\sec\theta+\tan\theta|\end{aligned}$$

Thus

$$\int \sec\theta d\theta = \log|\sec\theta+\tan\theta|$$

(∗ ∗ ∗) 19. Evaluate the integral. The attached table may be useful.

$$\int \frac{dx}{x\sqrt{9+4x^2}}$$

Solution: To evaluate this integral we recognize it as the form of a trigonometric substitution. Let $x = \frac{3}{2}\tan z$, and $dx = \frac{3}{2}\sec^2 z dz$. Note that we will have to evaluate the integral of csc along the way for this problem. You may use the attached table, but we will show the steps to evaluate the csc integral regardless.

$$\begin{aligned}\int \frac{\sec^2 z}{\tan z\sqrt{9+9\tan^2 z}}dz &= \frac{1}{3}\int \frac{\sec z}{\tan z}dz \\ &= \frac{1}{3}\int \frac{1}{\sin z}dz \\ &= \frac{1}{3}\int \frac{\sin z}{\sin^2 z}dz = \frac{1}{3}\int \frac{\sin z}{1-\cos^2 z}dz\end{aligned}$$

Now we have to set this up for a partial fraction decomposition: Let $\cos z = u$, and $du = -\sin z dz$

$$\begin{aligned}
-\frac{1}{3}\int \frac{1}{1-u^2}du &= -\frac{1}{6}\int \frac{1}{1+u} + \frac{1}{1-u}du \\
&= \frac{-1}{6}\log\left|\frac{1+u}{1-u}\right| \\
&= \frac{1}{6}\log\left|\frac{1-u}{1+u}\right| \\
&= \frac{1}{6}\log\left|\frac{1-\cos z}{1+\cos z}\right| \\
&= \frac{1}{6}\log\left|\frac{1-\cos z}{1+\cos z}\left(\frac{1-\cos z}{1-\cos z}\right)\right| \\
&= \frac{1}{6}\log\left|\frac{(1-\cos z)^2}{\sin^2 z}\right| \\
&= \frac{1}{3}\log\left|\frac{1-\cos z}{\sin z}\right| \\
&= \frac{1}{3}\log|\csc z - \cot z| + C \\
&= \frac{1}{3}\log\left|\frac{\sqrt{9+4x^2}-3}{x}\right| + C
\end{aligned}$$

Thus our final answer is

$$\int \frac{dx}{x\sqrt{9+4x^2}} = \frac{1}{3}\log\left|\frac{\sqrt{9+4x^2}-3}{x}\right| + C$$

(∗ ∗ ∗) 20. Evaluate the integral. The attached table may be useful.

$$\int \frac{x^2}{\sqrt{x^2-16}}dx$$

Solution: Let $x = 4\sec\theta$, so $dx = 4\sec\theta\tan\theta d\theta$. Then the integral is transformed to

$$\int \frac{x^2}{\sqrt{x^2-16}}dx = 16\int \sec^3\theta d\theta$$

Need to evaluate integral of $\sec^3 x$:

$$\int \sec^3\theta d\theta = \int \sec\theta\sec^2\theta d\theta$$

We will integrate this by parts: Let $\sec\theta = u$, so $\sec^2\theta = v'$ and $u' = \sec\theta\tan\theta$, $v = \tan\theta$.

$$\begin{aligned}\int \sec\theta\sec^2\theta d\theta &= \sec\theta\tan\theta - \int \tan^2\theta\sec\theta d\theta \\ &= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta d\theta \\ &= \sec\theta\tan\theta - \int \sec^3\theta d\theta + \int \sec\theta d\theta\end{aligned}$$

Now we move the integral of $\sec^3$ to the other side.

$$\begin{aligned}2\int \sec^3\theta d\theta &= \sec\theta\tan\theta + \int \sec\theta d\theta \\ &= \sec\theta\tan\theta + \log|\sec\theta + \tan\theta|\end{aligned}$$

Now dividing through by 2,

$$\int \sec^3\theta d\theta = \frac{1}{2}\left(\sec\theta\tan\theta + \log|\sec\theta + \tan\theta|\right) + C$$

Finally, multiplying by 16 to make it just like the original integral from the beginning of the solution,

$$\begin{aligned}16\int \sec^3\theta d\theta &= 8\sec\theta\tan\theta + 8\log|\sec\theta + \tan\theta| \\ &= \frac{1}{2}x\sqrt{x^2 - 16} + 8\log\left|x + \sqrt{x^2 - 16}\right| + C\end{aligned}$$

$$\int \frac{x^2}{\sqrt{x^2 - 16}}dx = \frac{1}{2}x\sqrt{x^2 - 16} + 8\log\left|x + \sqrt{x^2 - 16}\right| + C$$