

Mathematics 101 Quiz 5 Review Package – **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 15 pages, including 1 cover page and 23 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

Solutions posted at: <http://ubcengineers.ca/services/academic/tutoring/>

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.



Good Luck!

- (**) 1. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n^2 - 1}}$$

- (*) 2. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \sin\left(\frac{2n+1}{3-2n}\right)$$

- (*) 3. Determine if the following series converges or diverges.

$$\sum_{n=3}^{\infty} \sqrt{\frac{3e^n + 1}{e^n - 1}}$$

- (*) 4. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1}$$

- (*) 5. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{\log n}{n}$$

- (*) 6. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{\log n}$$

- (*) 7. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n}$$

- (*) 8. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2 + 1}}$$

(**) 9. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

(**) 10. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$$

(**) 11. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$$

(**) 12. Solve the following equation for c :

$$\sum_{n=2}^{\infty} (1+c)^{-n} = 2$$

(**) 13. Express the number $0.\overline{1234}$ as a ratio of two integers. You do not need to fully simplify your answer.

(**) 14. Express the number $1.35\overline{42}$ as a ratio of two integers. You do not need to fully simplify your answer.

(**) 15. For what real number(s) C does the following series converge?

$$\sum_{n=0}^{\infty} \left(\frac{n}{n^2 + 1} - \frac{C}{5n + 1} \right)$$

(**) 16. Find the limit of the following sequence:

$$\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right\}$$

(**) 17. Determine if the following series converges or diverges. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(**) 18. Determine how many terms of the series we need to sum before the difference between the partial sum and the sum of the series is $|\text{error}| < 0.0001$.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\log(n+1)}$$

- (**) 19. Determine how many terms of the series need to be summed in order for the partial sum to be correct to 3 decimal places.

$$\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{2^n}$$

(***) 20. Determine for which (if any) natural numbers k the following series converges.

$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^k}$$

(***) 21. Determine if the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$$

(***) 22. Find the sum of the series. Hint: Expand in terms of partial fractions.

$$\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$$

(***) 23. Consider the Fibonacci sequence defined by

$$f_1 = 1, \quad f_2 = 1, \quad f_n = f_{n-1} + f_{n-2}$$

(a) Prove that $\frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}$.

(b) Using the identity proved in part (a), find the sum of the series $\sum_{n=2}^{\infty} \frac{1}{f_{n-1}f_{n+1}}$.

(c) Again using the identity proved in part (a), find the sum of the series $\sum_{n=2}^{\infty} \frac{f_n}{f_{n-1}f_{n+1}}$.