Mathematics 152 Midterm 1 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 16 pages, including 1 cover page and 22 questions. Problems are ranked in difficulty as ($*$) for easy, ($**$) for medium, and ($***$) for difficult.

> Solutions posted at: `http://ubcengineers.ca/tutoring/`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources (All solutions prepared by the EUS.):

- Schuam's Outline of Matrix Operations; Richard Bronson

- Calculus 7th ed; James Stewart

- Linear Algebra; Sterling K. Berberian

- Linear Algebra and Its Applications 3rd ed; Gilbert Strang

- Linear Algebra and Matrix Theory; Robert Stoll

Want a warm up?	Short on study time?	Want a challenge?
These are the easier problems	These cover most of the material	These are some tougher questions
1, 2, 4, 5	6, 7, 8, 9, 11, 14	17, 18

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won't help you understand the material.

- **Sleep**—We have all been told we need 8 hours of sleep a night, university shouldn't change this. Get to know how much sleep you need and set up a regular sleep schedule.

E\US

(∗)   1. Let $\mathbf{A} = (3, 0, 2)$, $\mathbf{B} = (-4, 1, 6)$, $\mathbf{C} = (10, 9, 0)$, $\mathbf{D} = (7, 3, 5)$. Compute the following:

    (a) $2\mathbf{A} + 4\mathbf{D}$

    (b) $\|\mathbf{D}\|$

    (c) $\|\mathbf{B} - \mathbf{C}\|$

    (d) Compute the angle between $\mathbf{A}$ and $\mathbf{B}$

(∗) 2. Consider the augmented matrix $\begin{pmatrix} 1 & 2 & 6 \\ 3 & 6 & 7 \end{pmatrix}$. Determine if its associated linear system has one solution, no solutions, or infinitely many solutions.

(∗) 3. Consider the augmented matrix $\left(\begin{array}{ccc|c} 2 & 8 & 10 & 4 \\ 1 & 7 & 7 & 5 \\ 2 & 3 & 3 & 3 \end{array}\right)$. Determine whether the linear system associated with this matrix has one solution, no solutions, or infinitely many solutions.

(∗) 4. Consider the following lines of MATLAB code:
```
A = [1 0 0; 3 5 2; 2 3 4];
A = A + [3 2 1; 0 0 0; 1 2 1];
```
What will be the output if `A(2,1) + A(1,2)` is called?

(∗) 5. Find the projection of $(3, 5)$ onto the line $3x + 2y = 7$.

(∗∗)   6. Let $\mathbf{A} = (2, 1, 5)$, $\mathbf{B} = (-1, 5, -2)$, and $\mathbf{C} = (k, -3, 12)$.

    (a) For what value(s) of k will $\mathbf{A}, \mathbf{B}, \mathbf{C}$ form a linearly dependent set?

    (b) Find the area of the triangle spanned by $\mathbf{A}$ and $\mathbf{B}$

    (c) Now redefine $\mathbf{C} = (1, -3, 4)$. Find the volume of the parallelepiped spanned by $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$.

(∗) 7. Consider the linear system

$$\begin{cases} x + 2y + z = 1 \\ -x + 3z = 1 \\ x - y - 3z = 0 \end{cases}$$

(a) Write this system as an augmented matrix.

(b) Write the system to row echelon form

(c) Write the system in reduced row echelon form

(d) Find the solution to the system

(∗∗) 8. The line L passes through the points $(9, 0, 1)$ and $(7, 2, 3)$.

    (a)   i. Find a parametric equation for L.

          ii. Find an equation form of the line L.

    (b) The plane P has the equation $-x + y + z = 5$. Is the line L parallel to P, perpendicular to P, or neither?

    (c) The plane Q has the equation $2x - 2z = 1$. Is the line L parallel to Q, perpendicular to Q, or neither?

    (d) Find an equation for the plane that is perpendicular to L and passes through the point $(6, 2, 4)$

$(***)$ 9. Find the plane that passes through the points $(0, -2, 5)$ and $(-1, 3, 1)$ and is perpendicular to the plane $2z = 5x + 4y$.

($*$) 10. Solve the vector equation $\mathbf{a} = \mathbf{a} \times (1, 2, 3) + (13, 5, -6)$.

($**$) 11. Does $\{(2, 3), (-1, 2), (4, -7)\}$ form a linearly independent set?

Question 12

$(*)$ 12. Compute the rank of $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 0 & 2 \end{pmatrix}$

$(**)$ 13. Find a 2 by 3 system whose general solution is $\mathbf{x}(w) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

(∗) 14. Find the angles which the vector $\mathbf{A} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$ makes with each of the coordinate axes. You may leave your answer in calculator ready form.

(∗) 15. (a) Find the work done in moving an object along a vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ if the applied force is $\mathbf{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$

(b) Find the angle between the applied force and the displacement.

$(***)$ 16. Find the minimum distance between the point $(9, 0, -2)$ and the plane $z = 3x - 2y + 6$

($*$) 17. Write the general solution to the linear system associated with the following augmented matrix.

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 1 & 4 & 5 & 4 \end{array}\right)$$

($**$) 18. (a) Find an equation for the plane perpendicular to the vector $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ and passing through the terminal point of the vector $\mathbf{B} = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$

(b) Find the distance from the origin to the plane.

($\ast\ast$) 19. Find the minimum distance between the point $(3, 2, 6)$ and the line $\mathbf{r}(t) = (3t - 2)\mathbf{i} + t\mathbf{j} - (2t + 5)\mathbf{k}$. You may leave your answer in calculator ready form.

(∗∗) 20. Find the point of intersection between the line $\mathbf{r}(t) = (3t-2)\mathbf{i}+t\mathbf{j}-(2t+5)\mathbf{k}$ and the plane $z = 3x-2y+6$.

(∗) 21. Consider the following lines of Matlab code:

```
A = ones(5);
A(:,3) = [1 2 3 4 5]';
```

(a) What is `A`?

(b) What will the output be if we call `det(A)`?

(∗∗) 22. Consider the following lines of Matlab code:

```
x = 1:7;
y = 1:0.3:1.7;
```

   (a) What is `x`?

   (b) What is `y`?

   (c) If you call `sin(y)`, what will the output be? If this operation is defined, you may leave your answers in terms of trigonometric functions.

   (d) Is `cross(x,y)` defined?