Mathematics 152 Midterm 1 Review Package – **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 11 pages, including 1 cover page and 22 questions. Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(***)$ for difficult.

Solutions posted at: `http://ubcengineers.ca/tutoring/`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources (All solutions prepared by the EUS.):

- Schuam's Outline of Matrix Operations; Richard Bronson
- Calculus 7th ed; James Stewart
- Linear Algebra; Sterling K. Berberian
- Linear Algebra and Its Applications 3rd ed; Gilbert Strang
- Linear Algebra and Matrix Theory; Robert Stoll

Want a warm up? These are the easier problems	Short on study time? These cover most of the material	Want a challenge? These are some tougher questions
1, 2, 4, 5	6, 7, 8, 9, 11, 14	17, 18

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won't help you understand the material.
- **Sleep**—We have all been told we need 8 hours of sleep a night, university shouldn't change this. Get to know how much sleep you need and set up a regular sleep schedule.

(*) 1. Let $\mathbf{A} = (3, 0, 2)$, $\mathbf{B} = (-4, 1, 6)$, $\mathbf{C} = (10, 9, 0)$, $\mathbf{D} = (7, 3, 5)$. Compute the following:

(a) $2\mathbf{A} + 4\mathbf{D}$

(b) $\|\mathbf{D}\|$

(c) $\|\mathbf{B} - \mathbf{C}\|$

(d) Compute the angle between $\mathbf{A}$ and $\mathbf{B}$

Solution:

(a) We perform direct computation:

$$\begin{aligned} 2\mathbf{A} + 4\mathbf{B} &= 2\,(3, 0, 2) + 4\,(7, 3, 5) \\ &= (6 + 28, 0 + 12, 4 + 20) \\ &= (34, 12, 24) \end{aligned}$$

(b) Compute the magnitude by adding the components in quadrature:

$$\begin{aligned} \|\mathbf{D}\| &= \sqrt{7^2 + 3^2 + 5^2} \\ &= \sqrt{49 + 9 + 25} \\ &= \sqrt{83} \end{aligned}$$

(c) First take the difference of the two vectors, then take the norm.

$$\begin{aligned} \|\mathbf{B} - \mathbf{C}\| &= \|\,(-4 - 10, 1 - 9, 6 - 0)\,\| \\ &= \|\,(-14, -8, 6)\,\| \\ &= \sqrt{14^2 + 8^2 + 6^2} \\ &= \sqrt{196 + 64 + 36} \\ &= \sqrt{296} \\ &= 2\sqrt{74} \end{aligned}$$

(d) Since there are two representations of the dot product as $\mathbf{A} \cdot \mathbf{B} = A_1B_1 + A_2B_2 + A_3B_3 = \|\mathbf{A}\|\|\mathbf{B}\| \cos\theta$, we can solve for the angle:

$$\begin{aligned} \cos\theta &= \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} \\ &= \frac{(3, 0, 2) \cdot (-4, 1, 6)}{\sqrt{3^2 + 0^2 + 2^2}\sqrt{4^2 + 1^2 + 6^2}} \\ &= 0 \end{aligned}$$

Thus, $\mathbf{A}$ and $\mathbf{B}$ are perpendicular; the angle between them is $\theta = \pi/2$.

(*) 2. Consider the augmented matrix $\left(\begin{array}{cc|c} 1 & 2 & 6 \\ 3 & 6 & 7 \end{array}\right)$. Determine if its associated linear system has one solution, no solutions, or infinitely many solutions.

Solution:

Performing elimination, we obtain $\left(\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & 0 & -11 \end{array}\right)$. The last row corresponds to $0x + 0y = -11$, so $0 = -11$. This is impossible, so there are no solutions.

(∗) 3. Consider the augmented matrix $\left(\begin{array}{ccc|c} 2 & 8 & 10 & 4 \\ 1 & 7 & 7 & 5 \\ 2 & 3 & 3 & 3 \end{array}\right)$. Determine whether the linear system associated with this matrix has one solution, no solutions, or infinitely many solutions.

Solution:

Performing elimination,

$$\begin{aligned} \left(\begin{array}{ccc|c} 2 & 8 & 10 & 4 \\ 1 & 7 & 7 & 5 \\ 2 & 2 & 3 & 3 \end{array}\right) &\sim \left(\begin{array}{ccc|c} 1 & 4 & 5 & 2 \\ 1 & 7 & 7 & 5 \\ 2 & 2 & 3 & 3 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & 4 & 5 & 2 \\ 0 & 3 & 2 & 3 \\ 0 & -6 & -7 & -1 \end{array}\right) \\ &\sim \left(\begin{array}{ccc|c} 1 & 4 & 5 & 2 \\ 0 & 3 & 2 & 3 \\ 0 & 0 & -3 & 5 \end{array}\right) \end{aligned}$$

Since the rank of the matrix is 3, there is one unique solution to the associated linear system.

(∗) 4. Consider the following lines of MATLAB code:

```
A = [1 0 0; 3 5 2; 2 3 4];
A = A + [3 2 1; 0 0 0; 1 2 1];
```

What will be the output if `A(2,1) + A(1,2)` is called?

Solution:

This corresponds to the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 5 & 2 \\ 2 & 3 & 4 \end{pmatrix}$.

Then, when the second command is called, A is reassigned to

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 5 & 2 \\ 2 & 3 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 5 & 2 \\ 3 & 5 & 5 \end{pmatrix}$$

Thus the output will be

$$\texttt{A(2,1) + A(1,2)} = a_{21} + a_{12} = 3 + 2 = 5$$

(∗) 5. Find the projection of $(3, 5)$ onto the line $3x + 2y = 7$.

Solution: Given two vectors $\mathbf{a}$ and $\mathbf{b}$, the projection of $\mathbf{a}$ onto $\mathbf{b}$ is

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{b}\|^2}\mathbf{b}$$

We are projecting $(3,5)$ onto a line whose normal vector is $(3,2)$. A vector parallel to the line is then $(-2,3)$. Thus, we project $(3,5)$ onto $(-2,3)$. The projection is then

$$\frac{(3,5)\cdot(-2,3)}{2^2+3^3}(-2,3) = \frac{9}{13}(-2,3)$$

(**) 6. Let $\mathbf{A} = (2,1,5)$, $\mathbf{B} = (-1,5,-2)$, and $\mathbf{C} = (k,-3,12)$.

(a) For what value(s) of k will **A, B, C** form a linearly dependent set?

(b) Find the area of the triangle spanned by $\mathbf{A}$ and $\mathbf{B}$

(c) Now redefine $\mathbf{C} = (1,-3,4)$. Find the volume of the parallelepiped spanned by $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$.

Solution:

(a) We need to find some $a, b \in \mathbb{R}$ such that $a\mathbf{A}+b\mathbf{B} = \mathbf{C}$. This is equivalent to the vector equation

$$a\begin{pmatrix}2\\1\\5\end{pmatrix} + b\begin{pmatrix}-1\\5\\-2\end{pmatrix} = \begin{pmatrix}k\\-3\\12\end{pmatrix}$$

Solving the system formed by the last two rows,

$$\begin{cases} a+5b=-3 \\ 5a-2b=12 \end{cases}$$

we obtain $a=2$, $b=-1$. Thus,

$$2a-b=k=4-(-1)=5$$

so $k=5$ will cause the set to be linearly dependent.

(b) To compute the area of the triangle we take the magnitude of the cross product, then divide by 2. The magnitude of the cross product corresponds to the area of the parallelogram spanned by the two vectors, so to find the triangle we only need half of that. Computing the cross product,

$$\mathbf{A}\times\mathbf{B}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1 & 5\\ -1 & 5 & -2\end{vmatrix} = -27\mathbf{i}-\mathbf{j}+11\mathbf{k}$$

Then calculating the magnitude,

$$\|\mathbf{A}\times\mathbf{B}\| = \sqrt{27^2+1+11^2}$$

Thus the area is

$$\frac{\sqrt{27^2+1+11^2}}{2}$$

(c) The volume V of the parallelepiped spanned by the three vectors is simply the magnitude of their determinant.

$$V = \begin{vmatrix}2 & 1 & 5\\ -1 & 5 & -2\\ 1 & -3 & 4\end{vmatrix} = 20$$

(*) 7. Consider the linear system

$$\begin{cases} x+2y+z=1 \\ -x+3z=1 \\ x-y-3z=0 \end{cases}$$

(a) Write this system as an augmented matrix.

(b) Write the system to row echelon form

(c) Write the system in reduced row echelon form

(d) Find the solution to the system

Solution:

(a) Taking the coefficients and putting them in the matrix, and constants on the right of the line,

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & 0 & 3 & 1 \\ 1 & -1 & -3 & 0 \end{array}\right)$$

(b) Performing elimination until all of the elements below the diagonal, we obtain

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

(c) Continuing elimination until there are only ones on the diagonal, we obtain

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

(d) Thus, $x=2$, $y=-1$, $z=1$.

(**) 8. The line L passes through the points $(9, 0, 1)$ and $(7, 2, 3)$.

(a) i. Find a parametric equation for L.

ii. Find an equation form of the line L.

(b) The plane P has the equation $-x + y + z = 5$. Is the line L parallel to P, perpendicular to P, or neither?

(c) The plane Q has the equation $2x - 2z = 1$. Is the line L parallel to Q, perpendicular to Q, or neither?

(d) Find an equation for the plane that is perpendicular to L and passes through the point $(6, 2, 4)$

Solution:

(a) i. $\mathbf{r}(t) = (9, 0, 1) + t(2, -2, -2)$

ii. Eliminating t from the above equation, we obtain $z = 1 + y$, and $x = 9 - y$.

(b) The normal vector to the plane is $(-1, 1, 1)$, which is a multiple of $(2, -2, -2)$, so then the line is perpendicular to the plane.

(c) Dotting the normal vector of the plane with the direction vector of the line, we get $(2, -2, -2) \cdot (2, 0, -2) = 8$, so then it is neither perpendicular nor parallel.

(d) Perpendicular to L means that it will be of the form

$$(2, -2, -2) \cdot (x - 6, y - 2, z - 4) = 0$$

so an equation of the plane is

$$2(x - 6) - 2(y - 2) - 2(z - 4) = 0$$

(* * *) 9. Find the plane that passes through the points $(0, -2, 5)$ and $(-1, 3, 1)$ and is perpendicular to the plane $2z = 5x + 4y$.

Solution: First, we will find a displacement vector between the two points (0,-2,5) and (-1,3,1). These two points can be considered as vectors having position vectors $\mathbf{r}_1 = (0, -2, 5)$ and $\mathbf{r}_2 = (-1, 3, 1)$. The displacement vector from $\mathbf{r}_1$ to $\mathbf{r}_2$ is given by:

$$\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1 = (-1, 5, -4)$$

This vector $\mathbf{d}$ will be in the plane that we are looking for. Another vector in the plane that we are looking for is the normal vector to the given plane. This normal vector can be read off from the coefficients of (x, y, z) if $ax + by + cz = d$. Thus, moving all terms to one side we obtain $5x + 4y - 2z = 0$, which means a vector normal to the given plane is $(5, 4, -2)$. This vector will be *in* the plane that we are trying to find, so now we have a second vector in this plane. We need to find a normal vector to this plane, which can be done by taking the cross product of the two vectors which are in the plane. Let $\mathbf{n}$ be the vector normal to the plane that we are looking for. Then,

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 5 & -4 \\ 5 & 4 & -2 \end{vmatrix} = (6, -22, -29)$$

The equation for the plane will be

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

where (x_0, y_0, z_0) is a point in the plane, and (a, b, c) is the vector normal to the plane. Thus

$$6(x-0) - 22(y+2) - 29(z-5) = 0$$

$$6x - 22y - 29z = 44 - 145 = -101$$

Solution is:

$$6x - 22y - 29z = -101$$

(∗) 10. Solve the vector equation $\mathbf{a} = \mathbf{a} \times (1, 2, 3) + (13, 5, -6)$.

Solution: Let $\mathbf{a} = (a_1, a_2, a_3)$. Evaluating the cross product, we get the equation

$$(a_1, a_2, a_3) = (3a_2 - 2a_3, a_3 - 3a_1, 2a_1 - a_2) + (13, 5, -6)$$

This is three equations in three unknowns. That is,

$$\begin{cases} 3a_2 - 2a_3 + 13 = a_1 \\ a_3 - 3a_1 + 5 = a_2 \\ 2a_1 - a_2 - 6 = a_3 \end{cases}$$

Solving this system (by whatever method you choose) yields $(a_1, a_2, a_3) = (3, -2, 2)$

(∗∗) 11. Does $\{(2,3), (-1,2), (4,-7)\}$ form a linearly independent set?

Solution: No, a subset of $\mathbb{R}^2$ can have at most 2 linearly independent vectors.

Question 12

(∗) 12. Compute the rank of $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 0 & 2 \end{pmatrix}$

Solution: Performing elimination, we obtain

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and see that there are now only two independent rows. Note that there are also only two independent columns. Thus $\text{rank}(A) = 2$

(∗∗) 13. Find a 2 by 3 system whose general solution is $\mathbf{x}(w) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Solution: We see that $x_1 = 1 + w$, $x_2 = 1 + 2w$, and $x_3 = w$. Plugging x_3 for w into the other equations gives $x_1 = x_3 + 1$, and $x_2 = 1 + 2x_3$. This can be translated into the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -2 & 1 \end{array}\right)$$

This augmented matrix is the system that has solution

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + w \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

(∗) 14. Find the angles which the vector $\mathbf{A} = 3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$ makes with each of the coordinate axes. You may leave your answer in calculator ready form.

Solution: Let the angles with the x, y, z axes be α, β, γ respectively.

- $\mathbf{A} \cdot \mathbf{i} = 3 = |\mathbf{A}| \cos\alpha \to \cos\alpha = 3/7 \to \alpha = \arccos(3/7)$
- $\mathbf{A} \cdot \mathbf{j} = -6 = |\mathbf{A}| \cos\beta \to \cos\beta = -6/7 \to \beta = \arccos(-6/7)$
- $\mathbf{A} \cdot \mathbf{k} = 2 = |\mathbf{A}| \cos\gamma \to \cos\gamma = 2/7 \to \gamma = \arccos(2/7)$

(∗) 15. (a) Find the work done in moving an object along a vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ if the applied force is $\mathbf{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$

(b) Find the angle between the applied force and the displacement.

Solution:

(a) $W = \mathbf{r} \cdot \mathbf{F} = 6 - 2 + 5 = 9J$

(b) We can find the angle using the dot product: $\mathbf{r} \cdot \mathbf{F} = 9 = |\mathbf{r}| \cdot |\mathbf{F}| \cos\theta$. Dividing through by $|\mathbf{r}|$ and $|\mathbf{F}|$, then inverting the cosine, we have

$$\theta = \arccos\left(\frac{9}{\sqrt{6} \cdot \sqrt{38}}\right)$$

(∗ ∗ ∗) 16. Find the minimum distance between the point $(9, 0, -2)$ and the plane $z = 3x - 2y + 6$

Solution: First, find the parametric form of the plane. First find 3 points inside the plane by inspection. Three points could be $(0, 0, 6)$, $(0, 1, 4)$, $(1, 0, 9)$. It doesn't matter what these are actually chosen to be; all would give the same answer.

By subtracting points, we can then find 2 vectors which are in the plane. $\mathbf{u} = \mathbf{j}$ - $2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 3\mathbf{k}$. If you are not convinced by this, take $\mathbf{u} \times \mathbf{v}$ and see that it is indeed the normal vector of the original plane.

The parametric form of the plane is then $\mathbf{r}(s, t) = s\mathbf{i} + t\mathbf{j} + (6 - 2t + 3s)\mathbf{k}$ Then, the general vector between the point of interest $(9, 0, -2)$ and the plane is:

$$\mathbf{d}(s, t) = (s - 9)\mathbf{i} + t\mathbf{j} + (8 - 2t + 3s)\mathbf{k}$$

Since the minimum distance between the point and plane occurs if the point lies along the normal vector to the plane, let's project the vector $\mathbf{d}$ onto the normal vector of the plane, $\mathbf{n} = \langle 3, -2, -1\rangle$.

$$\begin{aligned} d &= \left|\frac{\mathbf{d}\cdot\mathbf{n}}{\mathbf{n}\cdot\mathbf{n}}\right| |\mathbf{n}| \\ &= \left|\frac{3s - 27 - 2t - 8 + 2t - 3s}{9 + 4 + 1}\right| \sqrt{9 + 4 + 1} \\ &= \frac{35}{\sqrt{14}} \end{aligned}$$

(∗) 17. Write the general solution to the linear system associated with the following augmented matrix.

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 1 & 4 & 5 & 4 \end{array}\right)$$

Solution: Performing elimination yields $\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 2 & 3 & 3 \end{array}\right)$. Thus, $\mathbf{x} = \begin{pmatrix} -2 + t \\ 3/2 - t \\ t \end{pmatrix}$

(∗∗) 18. (a) Find an equation for the plane perpendicular to the vector $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ and passing through the terminal point of the vector $\mathbf{B} = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$

(b) Find the distance from the origin to the plane.

Solution:

(a) The equation can be found by observing that the plane passes through the point $(1, 5, 3)$ with normal vector $\mathbf{A}$.

$$2(x - 1) + 3(y - 5) + 6(z - 3) = 0$$

(b) Let $\mathbf{a}$ be the vector from the origin to the plane. $\mathbf{a} = \mathrm{c}\mathbf{A}$ because the minimizing distance will occur when the vector from the origin is perpendicular to the plane. Plug the components of the vector $\mathbf{a}$ into the equation for the plane.

$$2(2c - 1) + 3(3c - 5) + 6(6c - 3) = 0$$

Solving for c:

$$c = 35/49 = 5/7$$

Then, plugging in this value of c we have

$$\|\mathbf{a}\| = c \cdot \|\mathbf{A}\| = 5$$

so the distance between the origin and the plane is 5.

(∗∗) 19. Find the minimum distance between the point $(3, 2, 6)$ and the line $\mathbf{r}(t) = (3t-2)\mathbf{i} + t\mathbf{j} - (2t+5)\mathbf{k}$. You may leave your answer in calculator ready form.

Solution: Form the general displacement vector $\mathbf{a}$ between the point in question and the line in question.

$\mathbf{a} = (3t - 2 - 3)\mathbf{i} + (t - 2)\mathbf{j} - (2t + 5 + 6)\mathbf{k}$

Assert that the vector $\mathbf{a}$ is perpendicular to the line, that is, the dot product is zero:

$$\mathbf{a} \cdot (3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$$

This leads to an equation in t, which can be solved as

$$t = \frac{-5}{14}$$

Evaluating the displacement vector $\mathbf{a}$ at the t we just found then taking the magnitude to find the length gives

$$\left\|\mathbf{a}\left(-\frac{5}{14}\right)\right\| = \frac{\sqrt{85^2 + 33^2 + 144^2}}{14}$$

(∗∗) 20. Find the point of intersection between the line $\mathbf{r}(t) = (3t-2)\mathbf{i} + t\mathbf{j} - (2t+5)\mathbf{k}$ and the plane $z = 3x - 2y + 6$.

Solution:
With the line

$$\mathbf{r}(t) = (3t - 2)\mathbf{i} + t\mathbf{j} - (2t + 5)\mathbf{k}$$

and plane

$$z = 3x - 2y + 6$$

we plug in the components of the line into the plane.
Thus $-2t - 5 = 3(3t - 2) - 2t + 6$. Solving for t gives $t = -5/9$.

We plug this value of t into the equation of the line

$$\mathbf{r}\left(-\frac{5}{9}\right) = -\frac{11}{3}\mathbf{i} - \frac{5}{9}\mathbf{j} - \frac{35}{9}\mathbf{k}$$

So the point of intersection is

$$\left(-\frac{11}{3}, -\frac{5}{9}, -\frac{35}{9}\right)$$

(∗) 21. Consider the following lines of Matlab code:

```
A = ones(5);
A(:,3) = [1 2 3 4 5]';
```

(a) What is `A`?

(b) What will the output be if we call `det(A)`?

Solution:

(a) The matrix `A` will be

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 5 & 1 & 1 \end{pmatrix}$$

(b) The determinant of A will be zero because it has repeated columns.

(∗∗) 22. Consider the following lines of Matlab code:

```
x = 1:7;
y = 1:0.3:1.7;
```

(a) What is `x`?

(b) What is `y`?

(c) If you call `sin(y)`, what will the output be? If this operation is defined, you may leave your answers in terms of trigonometric functions.

(d) Is `cross(x,y)` defined?

Solution:

(a) The notation means the vector starting with 1, then incrementing by 1 until 7. That is,

```
x = [1 2 3 4 5 6 7]
```

(b) The notation means the vector starting with 1, then incrementing by 0.3 until 1.7. Note that it will not surpass 1.7

```
y = [1 1.3 1.6]
```

(c) Calling `sin(y)` will apply the sin function to every entry in the vector. That is,

```
sin(y) = [sin(1) sin(1.3) sin(1.6)]
```

(d) The operation `cross(x,y)` is not defined because only `y` has three components. Since `x` has 7 components, it cannot enter into the cross product.