Mathematics 152 Midterm 2 Review Package –

UBC Engineering Undergraduate Society

Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(***)$ for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a $(*)$ problem.

Solutions posted at: `https://ubcengineers.ca/tutoring`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Matrix Operations; Richard Bronson
- Calculus 7th ed; James Stewart
- Linear Algebra; Sterling K. Berberian
- Linear Algebra and Its Applications 3rd ed; Gilbert Strang
- Linear Algebra and Matrix Theory; Robert Stoll

All solutions prepared by the EUS.

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb wont help you understand the material.
- **Sleep**—We have all been told we need 8 hours of sleep a night, university should not change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!

(∗) 1. Consider the linear system

$$\begin{cases} x + 2y + z = 1 \\ -x + 3z = 1 \\ x - y - 3z = 0 \end{cases}$$

(a) Write this system as an augmented matrix.

(b) Write the system to row echelon form

(c) Write the system in reduced row echelon form

(d) Find the solution to the system

(∗) 2. Compute the rank of $A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 0 & 2 \end{pmatrix}$

(∗) 3. (a) Find the work done in moving an object along a vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ if the applied force is $\mathbf{F} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$

(b) Find the angle between the applied force and the displacement.

(∗∗) 4. Consider the following lines of Matlab code:

```
x = 1:7;
y = 1:0.3:1.7;
```

(a) What is `x`?

(b) What is `y`?

(c) If you call `sin(y)`, what will the output be? If this operation is defined, you may leave your answers in terms of trigonometric functions.

(d) Is `cross(x,y)` defined?

(∗) 5. What matrix $A : \mathbb{R}^2 \to \mathbb{R}^2$ represents projection onto the x axis followed by projection onto the y axis?

(∗) 6. If $A = \begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ -2 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 3 & 1 \\ 2 & -2 & -2 \\ -1 & 2 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 3 & 1 & -3 \\ 0 & 2 & 6 \\ -1 & 2 & 1 \end{pmatrix}$

Compute

(a) AB

(b) AC

What can you say about AB and AC? What does it say about cancellation of matrices? Does $AB = AC$ imply that $B = C$?

(∗) 7. Compute the determinant of the matrix: $\begin{pmatrix} 2 & 6 & \log 2 & \pi^2 & e \\ 0 & 5 & 2 & 4 & \sqrt{5} \\ 0 & 0 & \pi & \sin(9) & 7 \\ 0 & 0 & 0 & -4 & 21 \\ 0 & 0 & 0 & 0 & 6 \end{pmatrix}$

(∗) 8. What matrix $A : \mathbb{R}^2 \to \mathbb{R}^2$ represents projection onto the x axis followed by projection onto the y axis?

(∗) 9. Compute the transpose of $A = \begin{pmatrix} -6 & 9 & 0 \\ 1 & -1 & 0 \\ 2 & \pi & 3 \\ 5 & 2 & 6 \end{pmatrix}$

(∗) 10. (a) Compute the product $A\mathbf{x} = \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(b) Without computing the determinant, determine if the matrix A is invertible or not.

(∗) 11. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 8 \\ 0 & -9 \end{pmatrix}$,

(a) If it is defined, compute AB

(b) If it is defined, compute BA

(∗) 12. Given $T(\mathbf{x}) = \begin{pmatrix} -1 & 3 \\ 9 & 4 \end{pmatrix}\mathbf{x}$, and $S(\mathbf{x}) = \begin{pmatrix} 3 & -2 & 6 \\ -4 & 6 & 2 \end{pmatrix}\mathbf{x}$, compute the following (if defined)

(a) $T \circ S$

(b) $S \circ T$

(c) $T\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right)$

(d) $S\left(\begin{pmatrix} -2 \\ 4 \end{pmatrix}\right)$

(e) $S\left(\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right)$

(∗) 13. (a) Find the matrix $R : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates vectors by 225°counterclockwise

(b) Find the image of $(2, 5)$ under this linear transformation.

(∗∗) 14. What matrix has the effect of rotating a vector $v \in \mathbb{R}^2$ through 90°clockwise, and then projecting the result onto the x axis?

(∗∗) 15. If possible, compute the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(∗) 16. Show that $A = A^{-1} = A^T$, if $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. What is the effect of A acting on a 3×3 matrix?

(∗∗) 17. If $T : \mathbb{R}^2 \to \mathbb{R}^2$, is a linear transformation, and we know that

$$T\left(\begin{pmatrix}2\\3\end{pmatrix}\right) = \begin{pmatrix}5\\7\end{pmatrix}, \quad T\left(\begin{pmatrix}1\\4\end{pmatrix}\right) = \begin{pmatrix}-2\\3\end{pmatrix}$$

(a) Compute $T\left(\begin{pmatrix}1\\-1\end{pmatrix}\right)$

(b) Find the matrix for the linear transformation T

(c) Find the inverse transformation T^{-1}

(∗∗) 18. If possible, compute the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 3 & 5 & 1 \\ 6 & 4 & 2 \end{pmatrix}$

(∗∗) 19. If A is an $n \times n$ matrix, and $\det(A) = x$, what are

(a) $\det(3A)$

(b) $\det(-A)$

(c) $\det(A^2)$

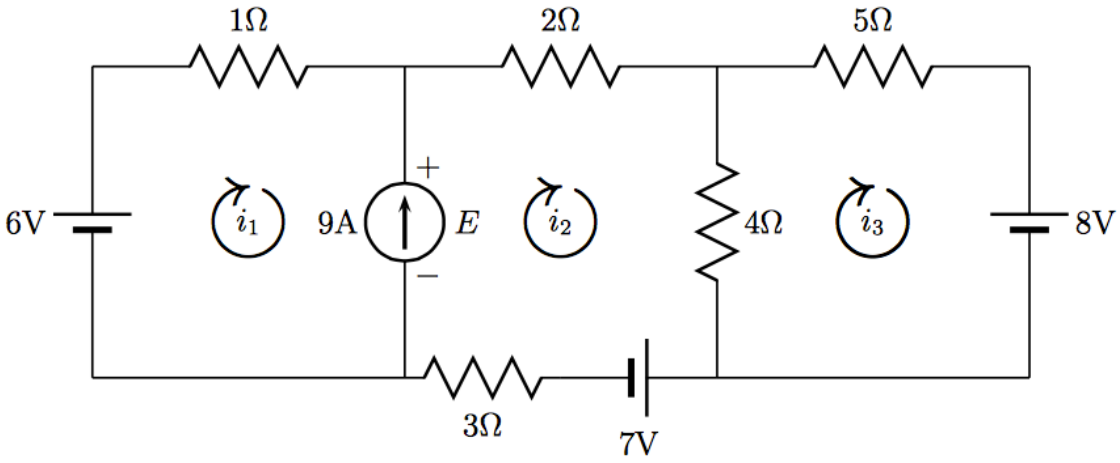
(d) $\det(A^{-1})$

(∗∗) 20. (a) Find the matrix $R : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects vectors across the line $y = -2x$.

(b) Show that $R^2 = I$.

(c) Reflect the vector $(-2, 3)$ across the line $y = -2x$.

(∗∗) 21. Set up the augmented matrix A corresponding to this resistor network, with the loop currents in the first three columns.



(∗∗) 22. If each year, 1/10 of electrical engineering students transfer to computer engineering, and 2/10 of computer engineering students transfer to electrical engineering, and there are initially 400 people in electrical engineering, and 600 people in computer engineering

(a) Find the transition matrix P

(b) Find how many students there are in each discipline after 2 years?

(∗∗) 23. A Physics 158 course is taught in two sections, and initially 400 students are in section 201, and 350 students are in section 203. If every week 1/4 of those in section 201 and 1/3 of those in section 203 permanently drop the course, and 1/6 of each section transfer to the other section,

(a) Find the transition matrix P

(b) the number of students in each state after 2 weeks.

You may leave your answer in calculator ready form. (That is, there is no need to multiply out or add fractions to common denominators)

(∗∗) 24. Given $A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 8 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 9 \end{pmatrix}$

(a) Evaluate $\det(A)$ by reducing the matrix to upper triangular form.

(b) Compute the determinants of

i. B

ii. C

iii. AB

iv. A^TA

v. C^T

(∗∗) 25. Consider the linear system for the unknowns x, y, and z.

$$\begin{aligned} 4x + 2y - 3z - 6 &= 0 \\ x - 4y + z + 4 &= 0 \\ -x + 2z - 2 &= 0 \end{aligned}$$

(a) Write the system in an augmented matrix.

(b) Perform row operations on the augmented matrix to change it to upper triangular form.

(c) Find the solution to the problem from above.

(∗) 26. Consider the resistor network below:

(a) Set up the augmented matrix A corresponding to this resistor network

(b) Solve the augmented matrix for the currents 1,2, and 3

(c) Find the voltage V_0