Mathematics 152 Midterm 3 Review Package –

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as ($*$) for easy, ($**$) for medium, and ($***$) for difficult.

Solutions posted at: **https://ubcengineers.ca/tutoring**

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Matrix Operations; Richard Bronson

- Calculus 7th ed; James Stewart

- Linear Algebra; Sterling K. Berberian

- Linear Algebra and Its Applications 3rd ed; Gilbert Strang

- Linear Algebra and Matrix Theory; Robert Stoll

Want a warm up?	Short on study time?	Want a challenge?
These are the easier problems	These cover most of the material	These are some tougher questions
1, 2, 3	7, 9, 10, 11	10, 11, 12

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won't help you understand the material.

- **Sleep**—We have all been told we need 8 hours of sleep a night, university should not change this. Get to know how much sleep you need and set up a regular sleep schedule.

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Good Luck!

(∗)   1. Compute the determinant of the matrix: $\begin{pmatrix} 2 & 6 & \log 2 & \pi^2 & e \\ 0 & 5 & 2 & 4 & \sqrt{5} \\ 0 & 0 & \pi & \sin(9) & 7 \\ 0 & 0 & 0 & -4 & 21 \\ 0 & 0 & 0 & 0 & 6 \end{pmatrix}$

(∗)   2. Compute the transpose of $A = \begin{pmatrix} -6 & 9 & 0 \\ 1 & -1 & 0 \\ 2 & \pi & 3 \\ 5 & 2 & 6 \end{pmatrix}$

(∗) 3. (a) Compute the product $A\mathbf{x} = \begin{pmatrix} 3 & -6 & 0 \\ 0 & 2 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

   (b) Without computing the determinant, determine if the matrix A is invertible or not.

(∗) 4. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & -5 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 8 \\ 0 & -9 \end{pmatrix}$,

   (a) If it is defined, compute AB

   (b) If it is defined, compute BA

(∗∗)   5. If possible, compute the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(∗)   6. Show that $A = A^{-1} = A^T$, if $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. What is the effect of A acting on a 3×3 matrix?

(∗∗)   7. If $T : \mathbb{R}^2 \to \mathbb{R}^2$, is a linear transformation, and we know that

$$T\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \quad T\left(\begin{pmatrix} 1 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

(a) Compute $T\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right)$

(b) Find the matrix for the linear transformation T

(c) Find the inverse transformation T^{-1}

($**$)   8. If possible, compute the inverse of the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 3 & 5 & 1 \\ 6 & 4 & 2 \end{pmatrix}$

(∗∗)   9. If A is an $n \times n$ matrix, and $\det(A) = x$, what are

    (a) $\det(3A)$

    (b) $\det(-A)$

    (c) $\det(A^2)$

    (d) $\det(A^{-1})$

(∗∗)   10. If each year, $1/10$ of electrical engineering students transfer to computer engineering, and $2/10$ of computer engineering students transfer to electrical engineering, and there are initially 400 people in electrical engineering, and 600 people in computer engineering

    (a) Find the transition matrix P

    (b) Find how many students there are in each discipline after 2 years?

($**$) 11. A Physics 158 course is taught in two sections, and initially 400 students are in section 201, and 350 students are in section 203. If every week 1/4 of those in section 201 and 1/3 of those in section 203 permanently drop the course, and 1/6 of each section transfer to the other section,

(a) Find the transition matrix P

(b) the number of students in each state after 2 weeks.

You may leave your answer in calculator ready form. (That is, there is no need to multiply out or add fractions to common denominators)

($**$) 12. Given $A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 8 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 4 & 6 \\ 1 & 5 & 9 \end{pmatrix}$

(a) Evaluate $\det(A)$ by reducing the matrix to upper triangular form.

(b) Compute the determinants of

    i. B

    ii. C

    iii. AB

    iv. $A^T A$

    v. C^T