Mathematics 101 Midterm Review Package

UBC Engineering Undergraduate Society

Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(***)$ for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a $(*)$ problem.

Solutions posted at: `http://ubcengineers.ca/tutoring/`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

The first 7 problems are review of high school material and are highly optional. They cover the basics of the different functions covered in high school.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

Want a warm up?	Short on study time?	Want a challenge?
These are the easier problems	These cover most of the material	These are some tougher questions
1, 2, 4, 5, 7, 12, 26	2, 6, 9, 10, 11, 14, 21, 24, 29	17, 18, 22, 25, 28, 29, 30

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb wont help you understand the material.
- **Sleep**—We have all been told we need 8 hours of sleep a night, university should not change this. Get to know how much sleep you need and set up a regular sleep schedule.

(∗) 1. Compute the value of the integral in terms of a.

$$\int_0^a \sqrt{a^2 - x^2}dx$$

(∗) 2. Suppose that

$$\int_{-2}^{0} f(x)dx = 5, \quad \int_0^3 f(x)dx = -7, \quad \int_1^{-2} g(x)dx = -3, \quad \int_1^3 g(x) = 1$$

Find the value of

$$\int_{-2}^{3} (f(x) - 2g(x))\, dx$$

(∗) 3. Compute the value of the integral.

$$\int_0^1 x\left(1-\sqrt{x}\right)^2 dx$$

(∗) 4. Approximate the area under the graph of the function $y = x^2$ on the interval $(1, 6)$ with 5 subdivisions using

(a) Right endpoints

(b) Left endpoints

(∗) 5. Evaluate the integral by using a limit of Riemann sums.

$$\int_0^4 -5x^2 + 7x - 2dx$$

The following summation formulas may be useful.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(2n+1)(n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(∗∗) 6. Consider the following Riemann sum:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sin^2\left(3+\frac{i\pi}{n}\right)\frac{\pi}{n}=\int_a^b f(x)dx$$

Express this as a definite integral in two different ways. That is, provide two different sets of $a, b, f(x)$ such that the equality holds true.

(∗) 7. Write the following Riemann sum as a definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\sin\left(\sqrt{\log\left(2+\frac{5i}{n}\right)}\right)$$

(∗∗) 8. Evaluate the integral.

$$\int \frac{e^{2x}}{1+e^{4x}}dx$$

(∗∗) 9. Evaluate the integral.

$$\int \frac{e^{x}-1}{e^{x}+1}dx$$

(∗∗) 10. Write the following Riemann sum as a definite integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{2i}{n} + \frac{i^2}{n^2}\right)$$

(∗∗) 11. Suppose you approximate the function $f(x) = \dfrac{x^4}{4} + x^2 + 5x - 7$ on the interval $[-1, 3]$ with a right endpoint Riemann sum. Will this approximation be an overestimate or an underestimate?

(∗) 12. Compute the derivative of $f(x)$.

$$f(x) = \int_3^{x^2+5x} \sin t dt$$

(∗) 13. If $x(t) = e^t \cos t$ and $y(t) = e^t \sin t$, compute the value of the integral.

$$\int_2^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

(∗∗) 14. Prove that

$$\int_0^{\pi/2} x \sin x dx \leq \frac{\pi^2}{8}$$

(∗∗) 15. Compute the derivative of $f(x)$.

$$f(x) = \int_{\log(7x^2-2x+5)}^{9} \sqrt[4]{t^3 - \sin t} dt$$

(∗∗) 16. Compute the derivative of $f(x)$.

$$f(x) = \int_{e^x}^{\sin(5x-9)} \arctan\left(\frac{1}{t}\right) dt$$

(∗ ∗ ∗) 17. Evaluate the integral

$$\int_a^b e^x dx$$

using a Riemann sum. You may use L'Hopital's rule to evaluate any limits.

Hint 1. The following formula may be useful.

$$\sum_{k=0}^{n-1} z^k = \frac{z^n - 1}{z - 1}$$

(∗ ∗ ∗) 18. (a) Let A_n be the area of a polygon with n equal sides inscribed in a circle of radius r. Show that

$$A_n = \frac{1}{2} n r^2 \sin\left(\frac{2\pi}{n}\right)$$

(b) Evaluate the limit

$$\lim_{n\to\infty} A_n$$

(∗∗) 19. Evaluate the integral

$$\int_0^{\pi} e^{\cos t} \sin(2t) dt$$

(∗∗) 20. Evaluate the integral

$$\int \cos\left(\sqrt{x}\right) dx$$

(∗∗) 21. Evaluate the integral.

$$\int \frac{1}{1-\sin(x/2)}dx$$

(∗ ∗ ∗) 22. Evaluate the integral.

$$\int \frac{\cos 2x}{\sin^2(2x) + 8} dx$$

(∗ ∗ ∗) 23. Evaluate the integral

$$\int x \arctan x dx$$

(∗∗) 24. Evaluate the integral.

$$\int (\sin x)(\sin 3x)dx$$

(∗ ∗ ∗) 25. Evaluate the integral.

$$\int \sqrt{1-\cos x}dx$$

(∗) 26. Find the volume of the solid generated by revolving the plane area bounded by $x - y - 7 = 0$, $x = 9 - y^2$ about the y-axis.

(∗) 27. Find the volume of the solid generated by revolving the plane area bounded by $y = x^2$ and $y = 4x - x^2$ around the line $y = 6$.

(∗ ∗ ∗) 28. Compute the area enclosed by the curve $y^2 = x^2 - x^4$.

Hint 2. *This curve is symmetric with respect to the x axis and the y axis. How can you use symmetry to help you calculate the area?*

(∗ ∗ ∗) 29. A solid has a base in the form of an ellipse with major diameter 10 and minor diameter 8. Find the volume if every section perpendicular to the major axis is an isosceles triangle with altitude 6.

(∗ ∗ ∗) 30. The base of a solid is the circle $x^2 + y^2 = 16x$, and every plane section perpendicular to the x-axis is a rectangle whose height is twice the distance of the plane of the section from the origin. Find the volume of this solid.