

# Mathematics 101 Quiz 4 Review Package – **Solution**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 15 pages, including 1 cover page and 20 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (\*) for easy, (\*\*) for medium, and (\*\*\*) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (\*) problem.

Solutions posted at: <http://ubcengineers.ca/services/academic/tutoring/>

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: [tutoring@ubcengineers.ca](mailto:tutoring@ubcengineers.ca). If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail [vpacademic@ubcengineers.ca](mailto:vpacademic@ubcengineers.ca).

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam's Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.



Good Luck!

- (\*) 1. Find the average value of  $f(x)$  on the interval  $[4, 8]$ .

$$f(x) = \frac{x}{\sqrt{x^2 - 15}}$$

- (\*) 2. Find the average value of  $f(x)$  on the interval  $[3, 4]$ .

$$f(x) = \frac{1}{25 - x^2}$$

- (\*) 3. If the following improper integral is convergent, evaluate it. Otherwise show that it is divergent.

$$\int_0^{\infty} \frac{dx}{x^2 + 4}$$

- (\*\*) 4. Find the average value of  $f(x)$  on the interval  $(2, 5)$ .

$$f(x) = \frac{1}{\sqrt{|x - 3|}}$$

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- (\*\*) 5. Find the centroid of the region bounded by the curves  $y = 4x - x^2$ ,  $y = x$ .

- (\*\*) 6. If the following improper integral is convergent, evaluate it. Otherwise show that it is divergent.

$$\int_0^3 \frac{dx}{\sqrt{9-x^2}}$$

- (\*\*) 7. Show that the integral converges.

$$\int_1^{\infty} e^{-x^2} dx$$

- (\*\*) 8. Determine if the integral converges or diverges.

$$\int_1^{\infty} \frac{1}{\sqrt{x^4 + 2x + 6}} dx$$

- (\*\*) 9. Determine if the integral converges or diverges.

$$\int_0^{\infty} \frac{x}{\sqrt{x^4 + 1}} dx$$

(\*\*) 10. Find the limit of the following sequence.

$$a_n = n - \sqrt{n+1}\sqrt{n+3}$$

(\*\*) 11. For what real number(s)  $C$  does the following integral converge?

$$\int_0^{\infty} \left( \frac{x}{x^2 + 4} - \frac{C}{2x + 3} \right)$$

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- (\*\*) 12. Find the centroid of the region bounded by  $9x^2 + 16y^2 = 144$  in the first quadrant.



- (\*\*) 13. A right circular cylindrical tank of radius 2 m and height 8 m is full of water. Find the work done in pumping the water to the top of the tank. Assume that the density of water is  $1000 \text{ kg/m}^3$ . You may assume that the gravitational field strength is  $g = 10 \text{ m/s}^2$ .

- (\*\*) 14. Find an implicit solution to the following differential equation

$$\frac{dy}{dx} = \frac{y \cos x}{1 + 2y^2}$$

- (\*\*) 15. A uniform 100 ft long cable weighing 5 lb/ft supports a safe weighing 500 lb. Find the work done in winding 80 ft of the cable onto a drum.

(\*\*) 16. Solve the following initial value problem explicitly in terms of  $y$ :

$$\frac{dy}{dx} = x^2y^2 + xy^2 + yx^2 + xy, \quad y(0) = 5$$

(\*\*) 17. Determine the limit of the following sequence. Express your answer in terms of  $p$ .

$$\left\{ \frac{1}{\sqrt[p]{n^p}} \right\}_{n=1}^{\infty}$$

(\*\*\*) 18. Determine if the following integral converges or diverges.

$$\int_0^1 \frac{1}{\sqrt{x} \log x} dx$$

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- (\*\*\*) 19. (a) How much work is done in filling an upright cylindrical tank of radius 3 ft and height 10 ft with liquid weighing  $w$  lb/ft<sup>3</sup> through a hole in the bottom?
- (b) How much if the tank is horizontal?

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(\*\*\*) 20. Find the area between the curve  $y^2 = \frac{x^2}{1-x^2}$  and its vertical asymptotes.