Physics 170 Midterm 1 Review Package – **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. If you're short on time, or looking for a challenge, see the tables further down this page for specific questions that you should attempt first. There is a formula sheet attached on the last page. This review package consists of 32 pages, including 1 cover page and 15 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as ($*$) for easy, ($**$) for medium, and ($***$) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a ($*$) problem.

Solutions posted at: `http://ubcengineers.ca/services/academic/tutoring/` If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail eus.ubc.academic@gmail.com.

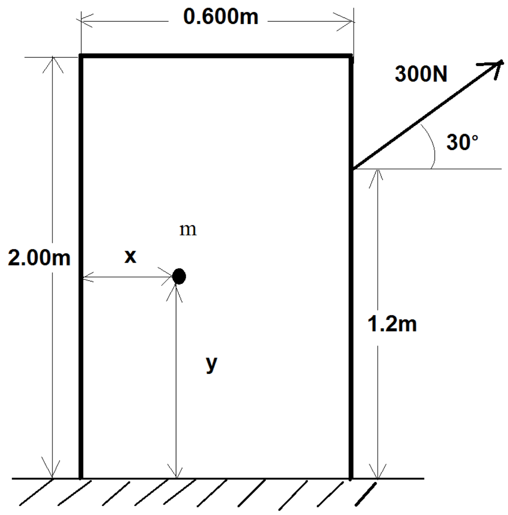
Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Fundamentals of Physics / David Halliday, Robert Resnick, Jearl Walker. – 9th ed.
- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.

All solutions prepared by the EUS.

Good Luck!

1. A 2.00m tall refrigerator of mass m has a static coefficient of friction $\mu s = 0.100$. When a pulling force of 300 N is applied as shown, the refrigerator barely slips and barely tips.
   (a) Find m and x.
   (b) With this information only, is it possible to find y? Why or why not?



Solution:

(a) The fact that the refrigerator "barely slips" and "barely tips" is key to providing enough information to make this problem solvable.

$$\Sigma F_x = 0$$
$$300cos(30) - F_f = 0$$
$$F_f = 300cos(30)N$$

$$\Sigma F_y = 0$$
$$300sin(30) + F_N - F_g$$
$$mg = F_N + 300sin(30)$$

Since the refrigerator is barely slipping, $F_N = F_f/\mu$

$$mg = 300(sin(30) + cos(30)/\mu)$$
$$m = \frac{300}{9.81}(sin(30) + \frac{cos(30)}{\mu}) = 280kg$$
$$F_g = 2748N$$

The clue "barely tips" means that the normal force is acting on the corner of the refrigerator under the applied force. To simplify the question, let's do the moment balance around this point.

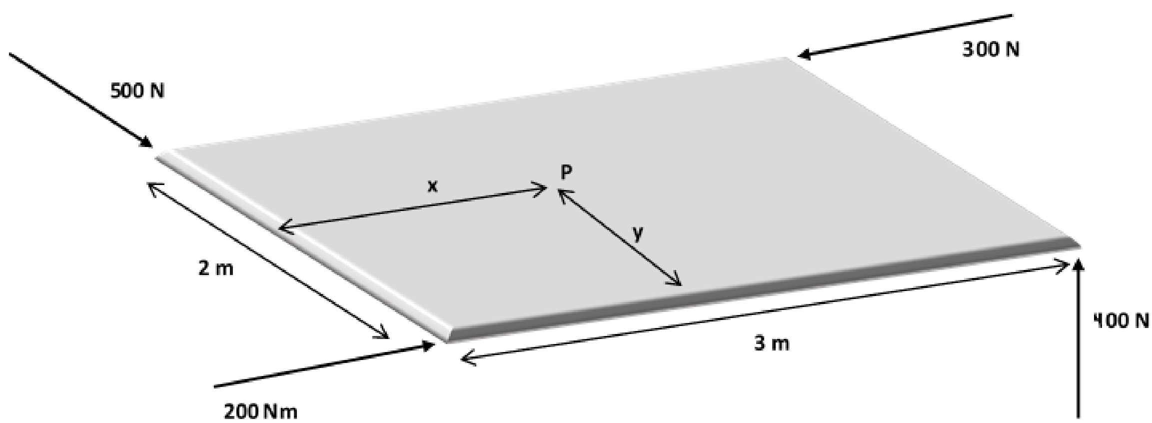
$$\Sigma M = 0$$

$$300cos(30) * 1.2 = 2748 * (0.6 - x)$$

$$x = 0.486m$$

(b) No, it is not possible to find y. Recall that a force can be moved anywhere along its line of action without consequence and F_g acts entirely in the y direction.

2. The diagram below shows a set of 3 forces and one moment acting on a rigid body.
   (a) Find the equivalent force and couple moment acting at point O.
   (b) Reduce all forces and moments to a single wrench acting on point P. Find the resulting force and moment vectors as well as the distances x and y.



Solution:

(a) Sum all forces and move to point O. Make sure to label your coordinate axes properly!

$$\mathbf{F_R} = -300i - 500j + 400k$$

$$\mathbf{M_{RO}} = 200i + (-300i \times 2j) + (-500j \times -3i) = 200i + 0j - 2100k$$

(b) To reduce the system to a wrench, the force and moment vector must be parallel. We can get this line from the resultant force vector since this vector does not change. The unit vector of the resultant force is:

$$u_{F_R} = -0.424i - 0.707j + 0.566k$$

Using the unknowns x and y to write the moment equations gives:

$$M_{Ri} = 200 - 400y = -0.424M_R$$

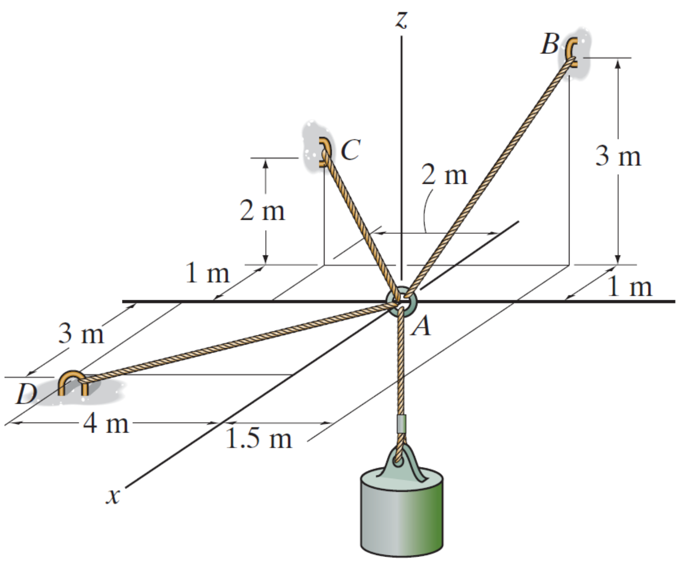
$$M_{Rj} = -400(3 - x) = -0.707M_R$$

$$M_{Rk} = 500x + 300(2 - y) = 0.566M_R$$

Solving this set of equations yields:

$$x = 1.05 \qquad y = 1.67 \qquad M_R = 1103Nm$$

3. The Diagram below shows a mass supported by three cables which are anchored to fixed supports.
   (a) Determine the tension in each of the three cables if the cylinder has a mass of 75 kg.
   (b) If each cable can withstand a maximum tension of 1000 N, determine the largest mass that this system can support.



Solution:

(a) Start by expressing each force as a magnitude along with a unit vector:

$$\mathbf{F_{AB}} = F_{AB}(-\frac{2}{7}i + \frac{3}{7}j + \frac{6}{7}k)$$

$$\mathbf{F_{AC}} = F_{AC}(-\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k)$$

$$\mathbf{F_{AD}} = F_{AD}(\frac{3}{5}i - \frac{4}{5}j)$$

$$\mathbf{W} = -735.75k$$

$$\Sigma F = 0, \mathbf{F_{AB}} + \mathbf{F_{AC}} + \mathbf{F_{AD}} + \mathbf{W} = 0$$

Equating the i,j,k components and solving for force magnitudes should yield:

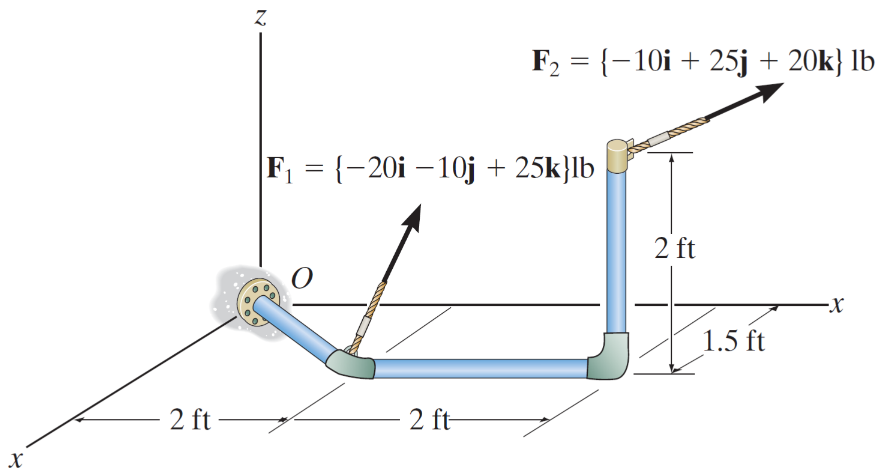
$$F_{AB} = 831\ N, F_{AC} = 35.6\ N, F_{AD} = 415\ N$$

(b) Based on the results of part a, we know the limiting case is when cable AB supports a load of 1000N.We could solve the entire system again but it is possible (and much simpler) to scale the results from a according to the new force in the cable.

$$\frac{F_{AB1}}{m_1} = \frac{F_{AB2}}{m_2}$$

$$m_2 = \frac{m_1 * F_{AB2}}{F_{AB1}} = 75 * 1000/831 = 90.3\ kg$$

4. Replace the two forces in the diagram below with a single force and couple moment acting at point O.



Solution: First find the resultant force:

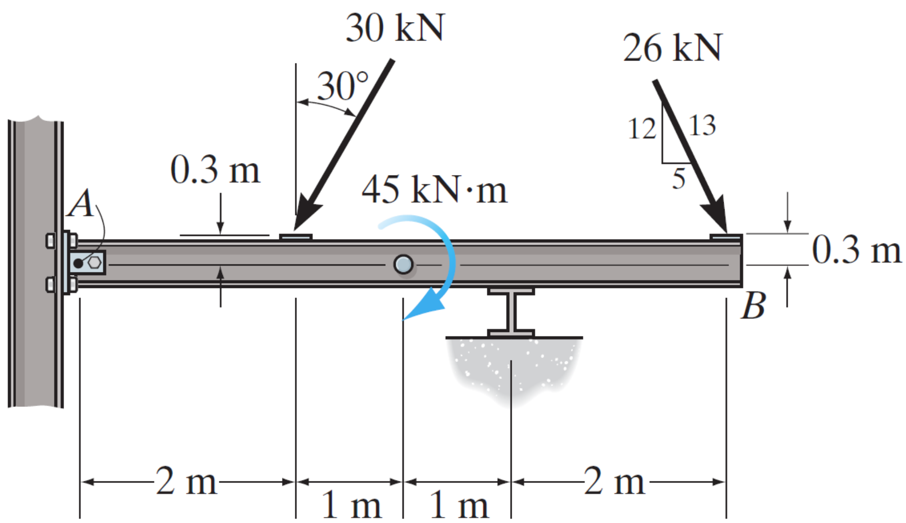
$$\mathbf{F_R} = (-20i - 10j + 25k) + (-10i + 25j + 20k) = -30i + 15j + 45k$$

To calculate the moment we need the vector distance from the origin to each force:

$$r_{O1} = 1.5i + 2j, r_{O2} = 1.5i + 4j + 2k$$

$$\mathbf{M_{RO}} = \begin{bmatrix} i & j & k \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 1.5 & 2 & 2 \\ -10 & 25 & 20 \end{bmatrix} = 80i - 87.5j + 102.5k$$

5. Consider the system below of a cantilevered beam with two forces and one couple moment acting on it.
   (a) Determine the equivalent force and moment acting at point A and the I beam.
   (b) Can the forces and couple moment acting on this beam be reduced to a single force? If so, determine this force and its location along the beam.
   (c) What conditions need to be met in order to reduce a system of forces and moments to a single force? (Hint: consider the wrench problem where the system can at most be reduced to a force and couple moment)



Solution:

(a) Calculate the resultant force as a magnitude and angle

$$F_x = 26(\frac{5}{13}) - 30sin(30) = -5kN$$

$$F_y = -26(\frac{12}{13}) - 30cos(30) = -50kN$$

$$F_R = \sqrt{5^2 + 50^2} = 50.2kN$$

$$\theta = tan^{-1}(\frac{50}{5}) = 84.3°$$

Sum moments around point A

$$M_{RA} = 30sin(30)\times0.3 - 30cos(30)\times2 - 26(\frac{5}{13})\times0.3 - 26(\frac{12}{13})\times6 - 45 = -239\ kNm = 239\ kNm\ CW$$

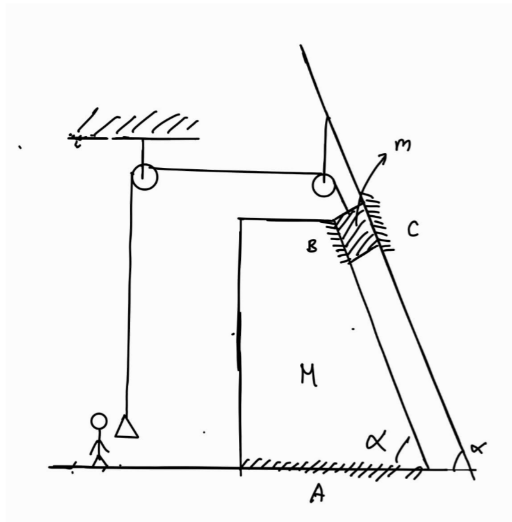
(b) Find the distance d along AB where the resultant force produces a moment around A equal to the initial moment.

$$\Sigma M_A = -50(d) = 30sin(30) \times 0.3 - 30cos(30) \times 2 - 26(\frac{5}{13}) \times 0.3 - 26(\frac{12}{13}) \times 6 - 45$$

$$d = 4.79\ m$$

(c) Forces must be coincident, coplanar, or perpendicular in order to be reduced to a single force with no moment. If this criteria is not met, the wrench is the most that the system can be simplified.

6. (**) **Stuck in a Gap**
A UBC Student had her bag stuck on a shelf while cleaning up her basement. To make things easier, they try to pull it out of the gap as shown in the picture below.



The bag has a mass m, and we model the shelf as a wedge with mass M. It is also known that the coefficients of static friction are $\mu_A = 0.2$, $\mu_B = 0.3$, and $\mu_C = 0.4$ for the wedge and floor, bag and wedge, and also between the bag and ceiling respectively.

(It is also given that $M = 10$ kg, $m = 1.5$ kg, and $\alpha = 60°$)

(a) Draw a large, clear, free body diagram!

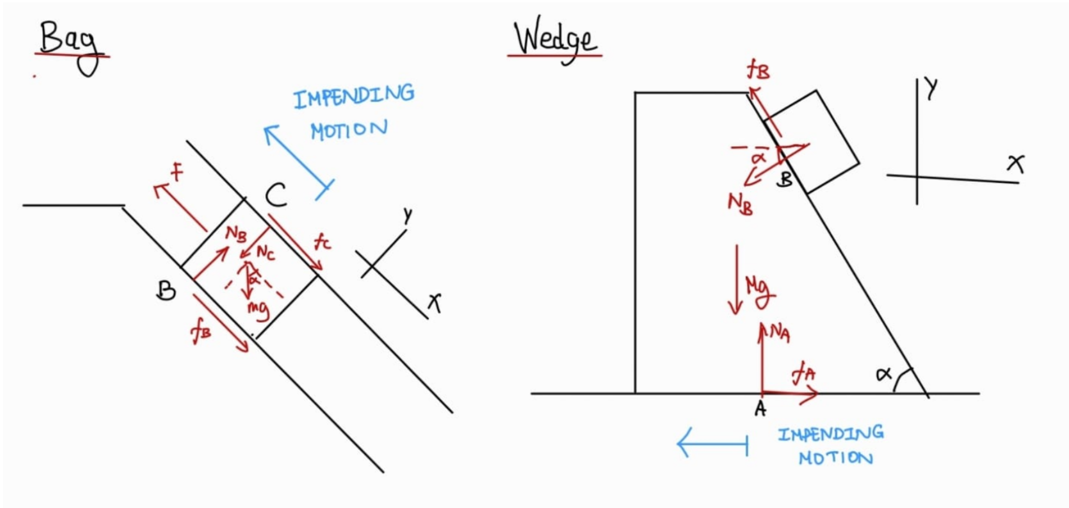
(b) Write down Cartesian equations for static equilibrium!

(c) List the unknown variables and determine the equations of impending motion!

(d) Find how much force must the student exert to pull out the bag.
(For extra challenge, try to solve the equations by hand)

Solution: Here is one approach to the problem:

(a) Draw a large, clear, free body diagram!



(b) Write down Cartesian equations for static equilibrium!

For the bag, we would have two equations for the X and Y axes:

$$\Sigma F_x = 0$$
$$F - f_B - f_C - mg\cos(\alpha) = 0 \tag{1}$$
$$\Sigma F_y = 0$$
$$N_B - mg\sin(\alpha) - N_C = 0 \tag{2}$$

Similarly for the wedge:

$$\Sigma F_x = 0$$
$$N_B\cos(\alpha) + f_B\sin(\alpha) - f_A = 0 \tag{3}$$
$$\Sigma F_y = 0$$
$$f_B\cos(\alpha) - N_B\sin(\alpha) - Mg + N_A = 0 \tag{4}$$

(c) List the unknown variables and determine the equations of impending motion!

From the free body diagram, the unknown variables are f_A, f_B, f_C, N_A, N_B, N_C, and F. We also have two equations for each body, so that leaves us with three more equations. There are three contact surfaces which means three fricitonal inequalities. Since we need three equations, the frictional inequalities will become:

$$f_A = \mu_A N_A \tag{5}$$
$$f_B = \mu_B N_B \tag{6}$$
$$f_C = \mu_C N_C. \tag{7}$$

(d) Find how much force must the student exert to pull out the bag!

By using a graphical calculator, we can obtain these solutions for the unknowns. Or we can solve it by hand.

Notice that with the impending motion equations (equations (5) to (7))), equations ((1) to (4)) would become:

$$F - \mu_B N_B - \mu_C N_C - mg\cos\alpha = 0 \tag{1}$$
$$N_B - mg\sin\alpha - N_C = 0 \tag{2}$$
$$N_B\cos\alpha + \mu_B N_B \sin\alpha - \mu_A N_A = 0 \tag{3}$$
$$\mu_B N_B\cos\alpha - N_B\sin\alpha - Mg + N_A = 0 \tag{4}$$

Solving the system (full working not shown) would give us:

$$F = \frac{Mg\mu_A(\mu_B + \mu_C)}{(1 + \mu_A\mu_C)\cos\alpha + (\mu_B - \mu_A)\sin\alpha}$$

Plugging in the numerical data gives us:

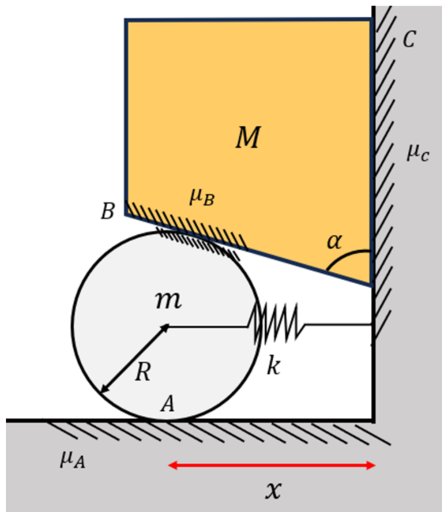
$$F = 24.5 \text{ N}$$

As a rule of thumb, always check the value of the other variables (working not shown). If the normal forces are negative, that means something is wrong with our assumption of the direction fo impending motion.

7. **(∗ ∗ ∗) A Peculiar Gun**

A new bullet launcher is modelled as the picture shown below, where the bullet is a cylindrical wedge (shown in yellow) with mass $M = 2$kg, $\alpha = 30°$, and has a circular part with mass $m = 20$kg. The spring has a relaxed length $x_0 = 2$m and $k = 10^4$ N/m. The bullet is pushed down slowly, and then released, The spring can elongate to a maximum length of x_{max} before the circular part slips upon releasing. If $\mu_A = 0.2$, $\mu_B = 0.4$, $\mu_C = 0.3$, find x_{max}!

For exam practice the next line can be ignored, but nonetheless to make solving the equations easier, it is provided that slipping would happen on surfaces A and C.

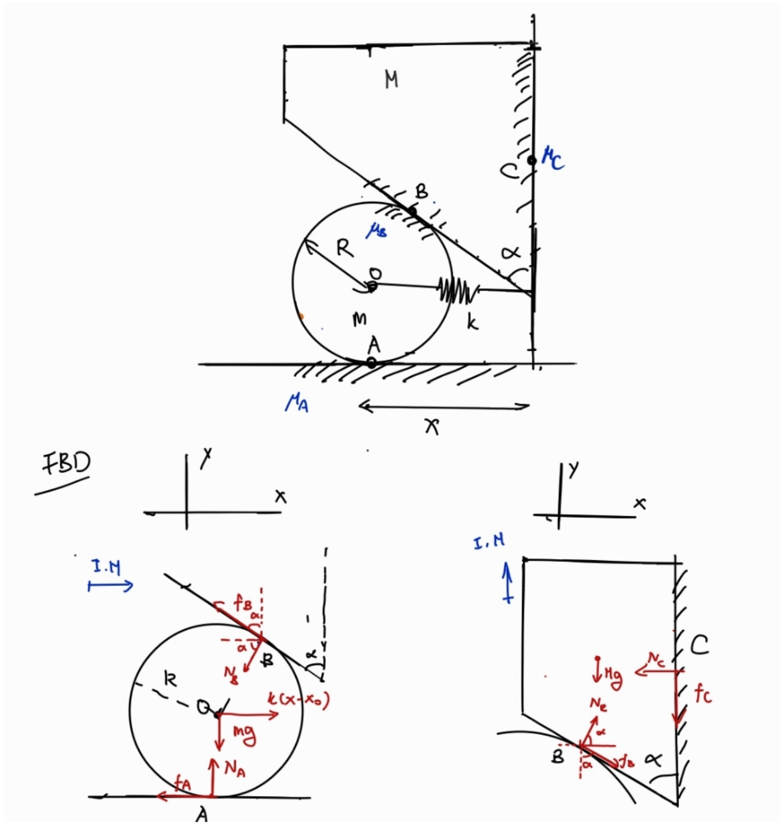


Solution:

Physical Reasoning:

As the bullet is pressed down, the circular part would "roll" to the left without slipping, increasing the length of the spring. The force on the spring would increase, up until the elastic forces would be strong enough to overcome the friction on surface C. We are trying to find the length of the spring at this specific condition.

We would start by constructing a free body diagram for the two bodies.



(I.M. here stands for impending motion)

From the free body diagram, there would be 7 unknowns: $f_A, N_A, f_B, N_B, f_C, N_C, x$. Then we would expect to get three equation of equilibrium from the circular bearing, two from the bullet. We also have three slipping conditions, from which we can pick two (already chosen) to include in our system of equations.

For the circular bearing (left) we would have:

$$\Sigma F_x = 0$$

$$k(x - x_B) - f_B \sin\alpha - N_B \cos\alpha = 0 \tag{1}$$

$$\Sigma F_y = 0$$

$$N_A - mg - N_B \sin\alpha + f_B \cos\alpha = 0 \tag{2}$$

$$\Sigma M_O = 0$$

$$-f_A R + f_B R = 0$$

$$f_A = f_B \tag{3}$$

And for the bullet we have:

$$\Sigma F_x = 0$$
$$N_B \cos\alpha + f_B \sin\alpha - N_C = 0 \tag{4}$$
$$\Sigma F_y = 0$$
$$N_B \sin\alpha - Mg - f_B \cos\alpha - f_C = 0 \tag{5}$$

And finally for the slipping conditions (assuming slipping occurs at surfaces A and C):

$$f_A = \mu_A N_A \tag{6}$$
$$f_B \leq \mu_B N_B \tag{7}$$
$$f_C = \mu_C N_C \tag{8}$$

Substituting f_A and f_C, we would have the following system of equations:

$$k(x - x_B) - f_B \sin\alpha - N_B \cos\alpha = 0 \tag{1}$$
$$N_A - mg - N_B \sin\alpha + f_B \cos\alpha = 0 \tag{2}$$
$$f_B = \mu_A N_A \tag{3}$$
$$N_B \cos\alpha + f_B \sin\alpha - N_C = 0 \tag{4}$$
$$N_B \sin\alpha - Mg - f_B \cos\alpha - \mu_C N_C = 0 \tag{5}$$

Solving these system of equations would give us:

$$N_A = 1.987 \text{ kN}$$
$$N_B = 4.270 \text{ kN}$$
$$N_C = 3.896 \text{ kN}$$
$$f_B = 397.4 \text{ N}$$
$$x = 2.40 \text{ m}$$

To make sure our assumptions are correct we must verify that our equation for impending motion for B still holds.

$$f_B \leq \mu_B N_B$$
$$397.4 \text{ N} \leq 1708 \text{ N}$$

Since the inequality is still satisfied, our assumptions are correct.

So we would obtain that the maximum spring length before slipping is $x_{max} = 2.40$ m.

8. **(∗) Telephone Pole**

A telephone pole (base located at the origin) is under 700 lbs of compression from three cables. The cables are attached to the top of the tower, 40ft above its base, and are securely connected to the ground at (10, 10, 10) ft, (10, -20, 10) ft, and (-15, 10, -3) ft. Determine the tension in each cable.

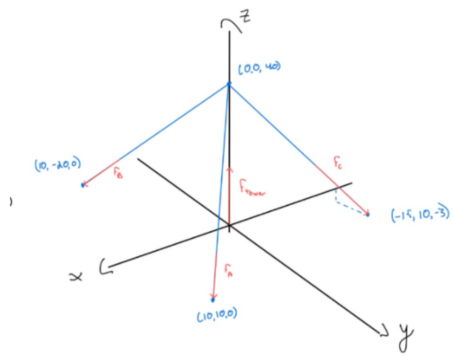
(a) Draw a large, clear, free-body diagram for the body!

(b) Determine Cartesian component force equations of equilibrium for the body!

(c) Determine the values of tension in all four cables!

Solution:

(a) Draw a large, clear, free-body diagram for the body!



(b) Determine Cartesian component force equations of equilibrium for the body!

We write the forces by expressing it as a magnitude along a unit vector:

$$\vec{F_A} = \frac{F_A}{42.43}(10\hat{i} + 10\hat{j} - 40\hat{k})$$
$$\vec{F_B} = \frac{F_B}{45.83}(10\hat{i} - 20\hat{j} - 40\hat{k})$$
$$\vec{F_C} = \frac{F_C}{46.63}(-15\hat{i} + 10\hat{j} - 43\hat{k})$$
$$\vec{F}_{tower} = 700\vec{k} \text{ lb}$$

To make calculations easier, we define:

$$A = \frac{F_A}{42.43} \qquad B = \frac{F_B}{45.83} \qquad C = \frac{F_C}{46.63}$$

For equilibrium the net force needs to be zero.

$$\vec{F_A} + \vec{F_B} + \vec{F_C} + \vec{F}_{pole} = 0$$

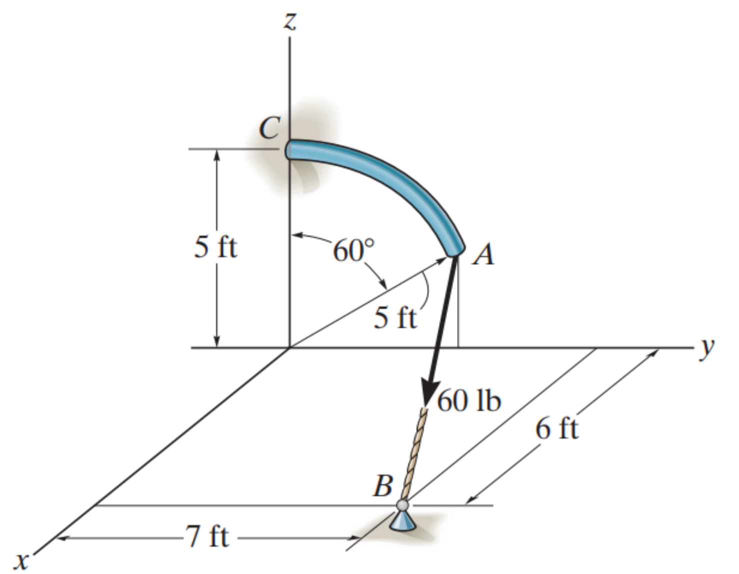
By analyzing each component:

$$F_x = 10A + 10B - 15C = 0$$
$$F_y = 10A - 20B + 10C = 0$$
$$F_z = -40A - 40B - 43C + 700 = 0$$

(c) Determine the values of tension in all three cables!
Solving would give us:

$$A = 4.531 \rightarrow F_A = 192 \text{ lb}$$
$$B = 5.663 \rightarrow F_B = 260 \text{ lb}$$
$$C = 6.796 \rightarrow F_A = 317 \text{ lb}$$

9. Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft to fail at support C. This requires a moment of $M = 85$ lb · ft to be developed at C.
Make sure to draw clear free body diagram of the system.



Solution:

(a) Skipped FBD in solution

(b) Position Vector CA:

$$\begin{aligned} r_{CA} &= r_A - r_C \\ &= 5\cos(30)\hat{j} + 5\sin(30)\hat{k} - 5\hat{k} \\ &= 4.33\hat{j} - 2.5\hat{k} \end{aligned}$$

Position Vector AB:

$$\begin{aligned} r_{AB} &= r_B - r_B \\ &= (6\hat{i} + 7\hat{j}) - (5\cos(30)\hat{j} + 5\sin(30)\hat{k}) \\ &= 6\hat{i} + 2.66\hat{j} - 2.5\hat{k} \end{aligned}$$

Unit Vector AB:

$$\begin{aligned} u_{AB} &= \frac{\vec{r_{AB}}}{|\vec{r_{AB}}|} \\ &= \left(\frac{6\hat{i} + 2.66\hat{j} - 2.5\hat{k}}{\sqrt{6^2 + 2.66^2 + (-2.5)^2}} \right) \\ &= 0.854\hat{i} + 0.38\hat{j} - 0.355\hat{k}F_{AB} \qquad\qquad = F(AB) \times u_{AB} \\ &= F_{AB} \times (0.854\hat{i} + 0.38\hat{j} - 0.355\hat{k}) \end{aligned}$$

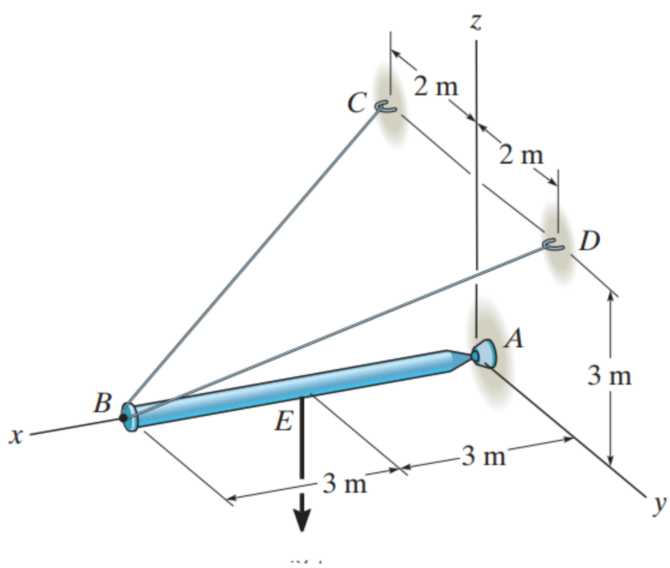
Moment about C which is along rope

$$
\begin{aligned}
M_C &= r_{CA} \times (\text{Fab}) \\
&= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4.33 & -2.5 \\ 0.854 & 0.38 & -0.355 \end{bmatrix} \\
85 &= F_{AB} \times \sqrt{(-0.5907)^2 + (2.135)^2 + (-3.697)^2} \\
85 &= F_{AB} \times 4.309
\end{aligned}
$$

Thus

$$F_{\text{AB}} = 19.722 \text{ lb}.$$

10. As shown in the diagram below, force of 600 N is applied on middle of the rod, point E.
    (a) Draw a large clear free body diagram of the given system.
    (b) Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.



Solution:

(a) FBD skipped.

(b) Calculate the force vector of T_{BC}:

$$\begin{aligned} T_{BC} &= T_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) \\ &= T_{BC}\left(\frac{-6\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{(-6)^2+(-2)^2+3^2}}\right) \\ &= -\frac{6}{7}T_{BC}\hat{\imath}-\frac{2}{7}T_{BC}\hat{\jmath}+\frac{3}{7}T_{BC}\hat{k} \end{aligned}$$

Calculate the force vector of T_{BD}:

$$\begin{aligned} T_{BD} &= T_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) \\ &= T_{BD}\left(\frac{-6\hat{\imath}-2\hat{\jmath}+3\hat{k}}{\sqrt{(-6)^2+(-2)^2+3^2}}\right) \\ &= -\frac{6}{7}T_{BD}\hat{\imath}-\frac{2}{7}T_{BD}\hat{\jmath}+\frac{3}{7}T_{BD}\hat{k} \end{aligned}$$

Apply equilibrium equations:

$$
\begin{aligned}
\Sigma M_z &= 0 \\
-6 \times \frac{2}{7} T_{BC} + 6 \times \frac{2}{7} T_B D &= 0 \\
\Sigma M_y &= 0 \\
-3 \times 600 + 6 \times \frac{3}{7} T_{BC} + 6 \times \frac{3}{7} T_{BD} &= 0 \\
T_{BC} &= 350 \text{ N} \\
T_{BD} &= 350 \text{ N} \\
\Sigma F_x &= 0 \\
A_x - \frac{6}{7} T_{BC} - \frac{6}{7} T_{BD} &= 0 \\
A_x - \frac{6}{7}(350) - \frac{6}{7}(350) &= 0 \\
A_x &= 600 \text{ N} \\
\Sigma F_y &= 0 \\
A_y - \frac{2}{7} T_{BC} + \frac{2}{7} T_{BD} &= 0 \\
A_y - \frac{2}{7}(350) + \frac{2}{7}(350) &= 0 \\
A_y &= 0 \text{ N} \\
\Sigma F_z &= 0 \\
A_z + \frac{3}{7} T_{BC} + \frac{3}{7} T_{BD} - 600 &= 0 \\
A_z + \frac{3}{7}(350) + \frac{3}{7}(350) - 600 &= 0 \\
A_z &= 300 \text{ N}
\end{aligned}
$$

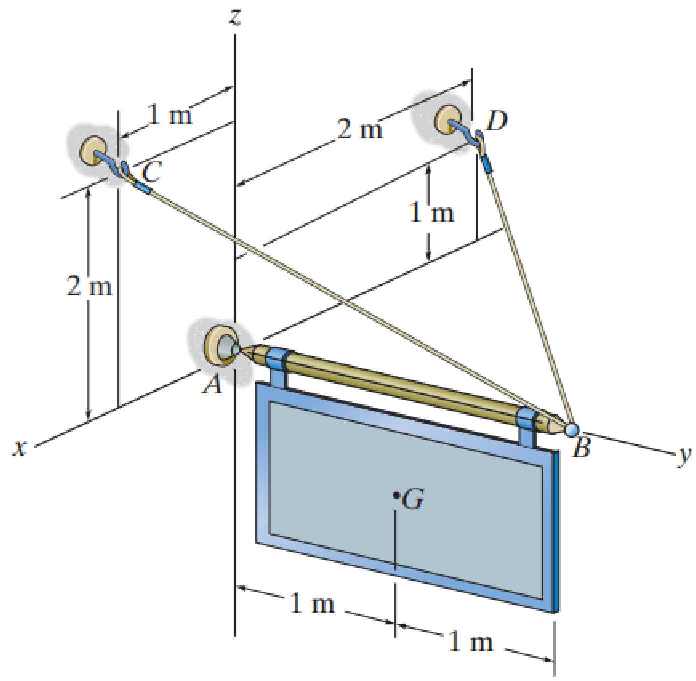
Thus

$$
\begin{aligned}
T_{BC} &= 350 \text{ N} \\
T_{BD} &= 350 \text{ N} \\
A_x &= 600 \text{ N} \\
A_y &= 0 \text{ N} \\
A_z &= 300 \text{ N.}
\end{aligned}
$$

11. The sign has a mass of 100 kg with center of mass at G.

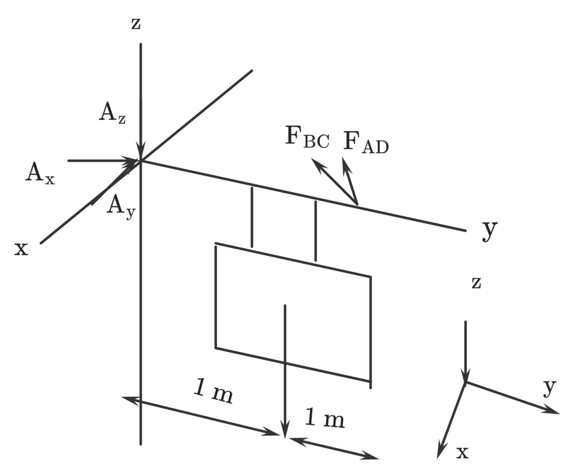
    (a) Draw clear and large free body diagram for given system.

    (b) Determine x, y, and z components of reaction at the ball-and-socket joint A and the tension in wires BC and BD.



Solution:

(a) FBD:



(b) Calculate the weight of the sign:

$$\begin{aligned} W &= mg \\ &= 100 \times 9.81 \\ &= 981 \text{ N} \end{aligned}$$

Force on cable BC:

$$\begin{aligned} F_{BC} &= F_{BC}\left(\frac{r_{BC}}{|r_{BC}|}\right) \\ &= F_{BC} \times \left(\frac{i - 2j + 2k}{\sqrt{1^2 + (-2)^2 + 2^2}}\right) \\ &= \frac{1}{3} \times F_{BC}\hat{i} - \frac{2}{3} \times F_{BC}\hat{j} + \frac{2}{3} \times F_{BC}\hat{k} \end{aligned}$$

Force on cable BD:

$$\begin{aligned} F_{BD} &= F_{BD}\left(\frac{r_{BD}}{|r_{BD}|}\right) \\ &= F_{BD} \times \left(\frac{-2i - 2j + k}{\sqrt{(-2)^2 + (-2)^2 + 1^2}}\right) \\ &= -\frac{2}{3} \times F_{BD}\hat{i} - \frac{2}{3} \times F_{BD}\hat{j} + \frac{1}{3} \times F_{BD}\hat{k} \end{aligned}$$

Now take moments:

$$\begin{aligned} \Sigma M_A &= 0 \\ r_B \times (F_{BC} + F_{BD} + W) &= 0 \\ 2\hat{j} \times \left(\left(\frac{1}{3}F_{BC}\hat{i} - \frac{2}{3}F_{BC}\hat{j} + \frac{2}{3}F_{BC}\hat{k}\right) + \left(-\frac{2}{3}F_{BD}\hat{i} - \frac{2}{3}F_{BD}\hat{j} + \frac{1}{3}F_{BD}\hat{k}\right)\right) + \hat{j} \times (-981\hat{k}) &= 0 \\ \left(\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981\right)\hat{i} + \left(-\frac{2}{3}F_{BC} + \frac{4}{3}F_{BD}\right)\hat{k} &= 0 \end{aligned}$$

$$\begin{aligned} \Sigma M_z &= 0 \\ -\frac{2}{3}F_{BC} + \frac{4}{3}F_{BD} &= 0 \\ F_{BC} &= 2F_{BD} \\ \Sigma M_x &= 0 \\ \frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981 &= 0 \\ \frac{4}{3} \times 2F_{BD} + \frac{2}{3}F_{BD} - 981 &= 0 \\ \frac{10}{3}F_{BD} &= 981 \\ F_{BD} &= 294.3 \text{ N} \\ F_{BC} &= 2F_{BD} \\ &= 2 \times 294.3 \\ &= 588.6 \text{ N} \end{aligned}$$

Now calculate reaction at A by equating force equation of equilibrium:

$$\Sigma F = 0$$
$$F_A + F_{BC} + F_{BD} + W = 0$$
$$\Sigma F_x = 0$$
$$A_x + \frac{1}{3} \times 588.6 - \frac{2}{3} \times 294.3 = 0$$
$$A_x = 0 \text{ N}$$
$$\Sigma F_y = 0$$
$$A_y - \frac{2}{3} \times 588.6 - \frac{2}{3} \times 294.3 = 0$$
$$A_y = 588.6 \text{ N}$$
$$\Sigma F_z = 0$$
$$A_z + \frac{2}{3} \times 588.6 + \frac{1}{3} \times 294.3 - 981 = 0$$
$$A_z = -392.4 - 98.1 + 981$$
$$A_z = 490.5 \text{ N}$$

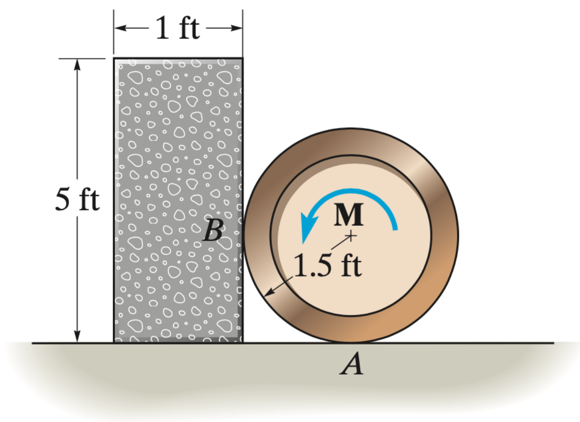
Thus,

$$F_{BD} = 294.3 \text{ N}$$
$$F_{BC} = 588.6 \text{ N}$$
$$A_x = 0 \text{ N}$$
$$A_y = 588.6 \text{ N}$$
$$A_z = 490.5 \text{ N}$$

12. The uniform concrete block has a weight of 300 lb. The coefficients of static friction are $\mu_A = 0.2$, $\mu_B = 0.3$, and between the concrete block and the floor, $\mu = 0.4$.

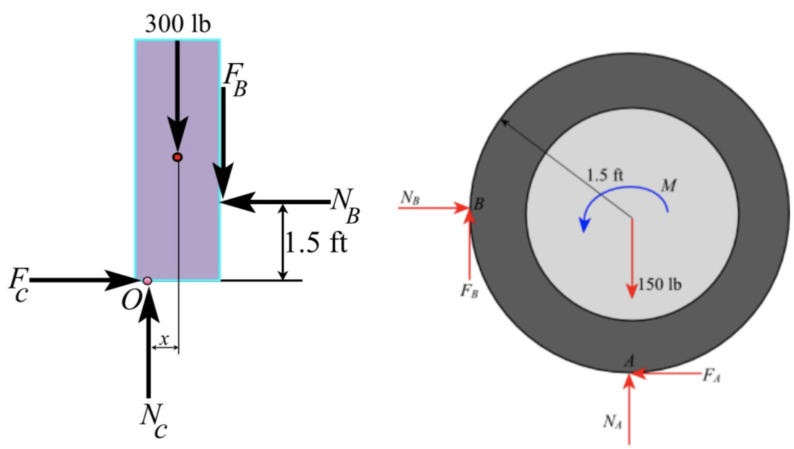
 (a) Draw big and clear free body diagram.

 (b) Determine the smallest couple moment that can be applied to the 150-lb wheel that will cause impending motion.



Solution:

(a) FBDs for concrete block and the wheel:



(b) Assume impending motion is cause on the wheel due to the rotation on the wheel, slipping ocurs at A and B.

$$F_A = \mu_A N_A \tag{12.1}$$
$$F_A = 0.2 N_A \quad \text{by substituting static friction coefficient at A} \tag{12.2}$$
$$F_B = \mu_B N_B \tag{12.3}$$
$$F_B = 0.3 N_B \quad \text{by substituting static friction coefficient at B} \tag{12.4}$$

Consider the equilibrium forces along horizontal direction.

$$\Sigma F_x = 0 \tag{12.5}$$
$$F_C - N_B = 0 \tag{12.6}$$

Now consider the equilibrium forces along vertical direction.

$$\Sigma F_y = 0 \tag{12.7}$$
$$N_C - F_B - W_C = 0 \tag{12.8}$$
$$N_C - 0.3N_B = 300 \tag{12.9}$$

Now calculate the moment about the point O.

$$\Sigma M_O = 0 \tag{12.10}$$
$$N_B(1.5\text{ft}) - (W_C)x - F_B(0.5 + x) = 0 \tag{12.11}$$
$$N_B(1.5\text{ft}) - (300\text{lb})x - F_B(0.5 + x) = 0 \tag{12.12}$$
$$1.35N_B - (300 + 0.3N_B)x = 0 \tag{12.13}$$

Again consider the equilibrium forces along horizontal direction.

$$\Sigma F_x = 0 \tag{12.14}$$
$$N_B - F_A = 0 \tag{12.15}$$
$$N_B = F_A \tag{12.16}$$
$$N_B = 0.2N_A \tag{12.17}$$

Now consider the equilibrium forces along vertical direction.

$$\Sigma F_y = 0 \tag{12.18}$$
$$N_A + F_B - 150 = 0 \tag{12.19}$$
$$N_A + 0.3N_B - 150 = 0 \tag{12.20}$$
$$N_A + 0.3N_B = 150 \tag{12.21}$$

Solve equations (17) and (21).

$$N_A + 0.3(0.2N_A) = 150 \tag{12.22}$$
$$1.06N_A = 150 \tag{12.23}$$
$$N_A = 141.509lb \tag{12.24}$$
$$N_B = 0.2(N_A) \tag{12.25}$$
$$= 0.2(141.509) \tag{12.26}$$
$$= 28.3018\text{lb} \tag{12.27}$$

Calculate the moment about the point A.

$$\Sigma M_A = 0 \tag{12.28}$$
$$M - N_B(1.5\text{ft}) - F_B(1.5\text{ft}) = 0 \tag{12.29}$$
$$M - N_B(1.5\text{ft}) - F_B(1.5\text{ft}) = 0 \tag{12.30}$$
$$M - N_B(1.5\text{ft}) - 0.3N_B(1.5\text{ft}) = 0 \tag{12.31}$$
$$M = 1.95N_B \tag{12.32}$$
$$= 1.95(28.3018) \tag{12.33}$$
$$= 55.19\text{lb} \cdot \text{ft} \tag{12.34}$$

Thus, the couple moment that can be applied to the wheel is 55.19lb · ft. Substitute 28.3018 lb for N_B in equation (6).

$$F_C - N_B = 0 \tag{12.35}$$
$$F_C - 28.3018 = 0 \tag{12.36}$$
$$F_C = 28.3018\text{lb} \tag{12.37}$$

Now substitute 28.3018 lb for N_B in equation (9).

$$N_C - 0.3N_B = 300 \tag{12.38}$$
$$N_C - 0.3(28.3018) = 300 \tag{12.39}$$
$$N_C = 308.49\text{lb} \tag{12.40}$$

Then substitute 28.3018 lb for N_B in equation (13).

$$1.35(28.3018) - (300 + 0.3(28.3018))x = 0 \tag{12.41}$$
$$308.49x = 38.207 \tag{12.42}$$
$$x = 0.12385 \text{ ft} \tag{12.43}$$

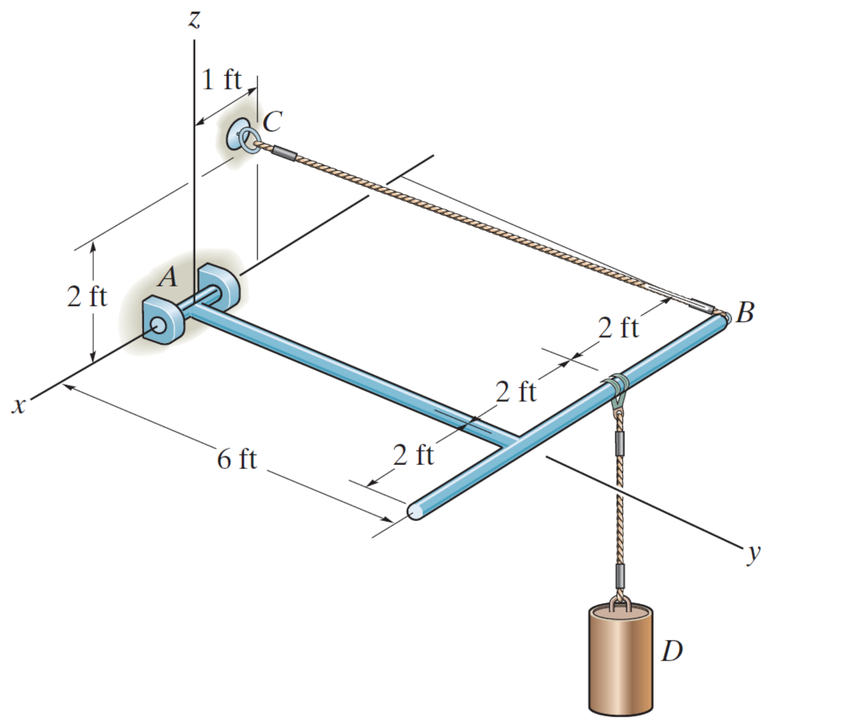
Calculate the maximum static friction on the block at the point C.

$$\begin{aligned}(F_C)_{\max} &= \mu_C N_C \\ &= 0.4(308.49) \\ &= 123.40 \text{ lb}\end{aligned}$$

So notice that $F_C < (F_C)_{\max}$.
Thus, the concrete block will not slip or tip at $x < 0.5$ ft.
The normal reaction at the point A is positive, thus the wheel will be in contact with the floor and our assumption is correct. Thus, the couple moment that can be applied to the wheel is

$$M = 55.19 \text{ lb} \cdot \text{ft}.$$

13. The member is supported by a pin at A and a cable BC. If the load at D is 300 lb determine the x, y, z components of reaction at the pin A and the tension in cable BC.



Solution:

(a) First write down the tension in vector form

$$\mathbf{T_{BC}} = T_{BC}(\frac{3}{7}i - \frac{6}{7}j + \frac{2}{7}k)$$

$$\Sigma F_x = 0 = A_x + \frac{3}{7}T_{BC}$$

$$\Sigma F_y = 0 = A_y - \frac{6}{7}T_{BC}$$

$$\Sigma F_z = 0 = A_z + \frac{2}{7}T_{BC} - 300$$

$$\Sigma M_x = 0 = -300(6) + (6)\frac{2}{7}T_{BC}$$

$$\Sigma M_y = 0 = M_{Ay} - 300(2) + (4)\frac{2}{7}T_{BC}$$

$$\Sigma M_z = 0 = M_{Az} - 300(6) - (6)\frac{3}{7}T_{BC} + (4)\frac{6}{7}T_{BC}$$

Solving this system of equations gives:

$$T_{BC} = 1050\ lb$$

$$A_x = -450\ lb$$

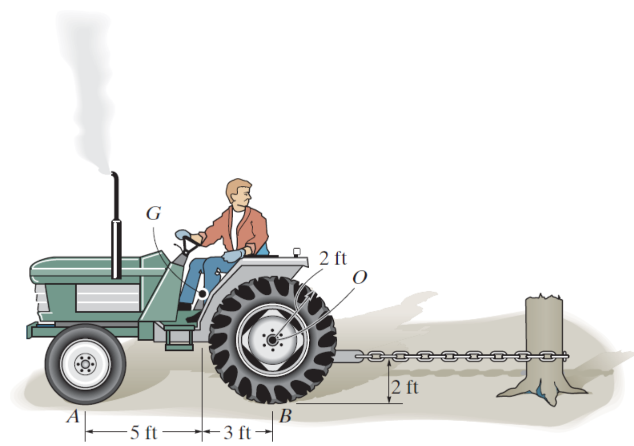
$$A_y = 900\ lb$$

$$A_z = 0$$

$$M_{Ay} = -600\ lb \cdot ft$$

$$M_{Az} = -900\ lb \cdot ft$$

14. The tractor pulls on the fixed tree stump as shown below. The front wheels are free to roll. The tractor weighs 3500 lb and has a center of gravity at G. The coefficient of static friction between the rear wheels and the ground is $\mu_s = 0.5$.
    (a) Determine the torque that must be applied by the engine to the rear wheels to cause them to slip.
    (b) What coefficient of friction between the rear wheel and the ground is required for the front wheel to lift before the rear wheel slips?



Solution:

(a) Although it is implied by the question, we should double check to make sure that the rear wheel slips before the front wheel lifts.

$$\Sigma M_a = 0 = -3500(5) - (2)T + (8)N_B$$
$$\Sigma F_y = 0 = -3500 + N_B + N_A$$
$$\Sigma F_x = 0 = T - (0.5)N_B$$

Solving this system of equations gives:

$$N_A = 1000\ lb \qquad N_B = 2500\ lb \qquad T = 1250\ lb$$

If there is a normal force at point A then the front wheels must be on the ground and our assumption is good. To find the torque on the wheel we sum the moments acting around the center of the wheel.

$$\Sigma M = 0 = (2)(0.5)N_B - \tau$$
$$\tau = 2500\ lb \cdot ft$$

(b) At the moment when the front wheel lifts, the normal force on the front wheel must be zero. Adjusting our equations from part (a) gives:

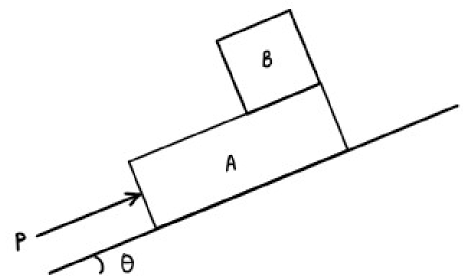
$$\Sigma M_a = 0 = -3500(5) - (2)T + (8)N_B$$

$$\Sigma F_y = 0 = -3500 + N_B$$

$$\Sigma F_x = 0 = T - (\mu)N_B$$

$$\mu = 1.5$$

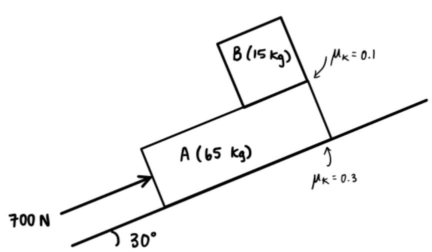
15. The diagram below shows block A on an inclined plane with block B on block A. Block A is acted upon by a force P parallel to the plane. The mass of block A is 65 kg. The mass of block B is 15 kg. The magnitude of P is 700N. μ_k between A and B is 0.10 and μ_k between A and the plane is 0.30. The angle of the inclined plane is 30^o.

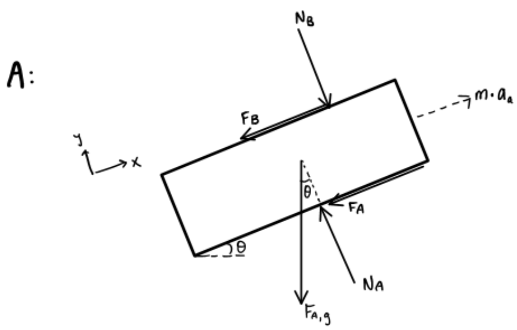


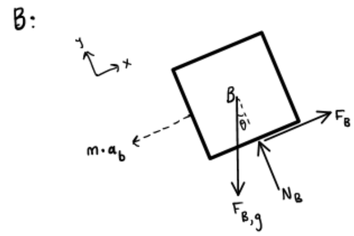
(a) Draw the free-body diagram for each block
(b) Determine the equations of motion for each block
(c) Solve these equations to determine the normal force of block A on block B, the normal force of the inclined plane on block A, and the acceleration of each block

Solution:

(a) FBD:







(b) When writing the equations of motion keep in mind that this is not in equilibrium and we must use F = ma

$$\Sigma F_A x = (65)a_A = 700 - (0.3)N_A - (0.1)N_B - (65)(g)\sin 30$$

$$\Sigma F_A y = 0 = N_A - N_B - (65)(g)\cos 30$$

$$\Sigma F_B x = -(15)a_B = (0.1)N_B - (15)(g)\sin 30$$

$$\Sigma F_B y = 0 = N_B - (15)(g)\cos 30$$

(c) Solving these equations yields:

$$N_B = 127\ N \qquad a_B = 4.06\ m/s \qquad N_A = 680\ N \qquad a_A = 2.53\ m/s$$