Physics 157 Midterm 1 Review Package - **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as ($*$) for easy, ($**$) for medium, and ($***$) for difficult. Difficulty is subjective, so do not be discouraged if you are stuck on a ($*$) problem.

> Solutions will be posted at: `https://ubcengineers.ca/review-packages`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Want a warm up?	Short on study time?	Want a challenge?
These are the easier problems	These cover most of the material	These are some tougher questions
1, 2, 3, 4	6, 9, 11, 12, 15	14, 15, 16

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Fundamentals of Physics / David Halliday, Robert Resnick, Jearl Walker. – 9th ed.

- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.

All solutions prepared by the EUS.

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.

- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb wont help you understand the material.

- **Sleep**—Weve all been told we need 8 hours of sleep a night, university shouldnt change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!

(∗) 1. A cylindrical aluminum rod, with an initial length of $0.8000\,\text{m}$ and a radius of $1000.0\,\mu\text{m}$, is clamped in place at one end and then stretched by a machine pulling parallel to its length at its other end. Assume that the rod's mass density does not change and that its radius decreases uniformly along the length of the rod. Find the magnitude of the force required to decrease the radius to $999.9\,\mu\text{m}$. The Young's modulus for aluminum is $Y_{Al} = 70\,\text{GPa}$.

Solution:

Let $l = 0.8\,\text{m}$ be the initial length of the rod and $r = 0.001\,\text{m}$ be the initial radius of the rod.

Let l' be the length of the rod after stretching and $r' = 0.0009999\,\text{m}$ the radius of the rod after stretching.

The volume of the rod will stay constant, thus

$$V = lr^2\pi = l'(r')^2\pi$$

We solve, using our numbers above, for $l' = 0.80016\,\text{m}$.

We find $\Delta l = l' - l = 0.00016\,\text{m}$, and $A' = \pi(r')^2 = 3.1410\,\text{mm}^2$ and plug the numbers into Hooke's law

$$\frac{F}{A'} = Y\frac{\Delta l}{l}$$

This yields

$$F = 43.973\,\text{N}$$

($*$) 2. A cylinder with a leak-less, frictionless piston contains $1.0\,\text{m}^3$ of a monatomic gas with a gauge pressure of $1\,\text{atm}$. The gas is very slowly compressed until the final volume is only $0.4\,\text{m}^3$. How much work W must be done to accomplish this compression?

Solution:
In the ideal gas law, P stands for absolute pressure. We can take our atmospheric pressure to be $1\,\text{atm}$, and since the gauge pressure is $1\,\text{atm}$, we know that absolute pressure $P_0 = 1\,\text{atm} + 1\,\text{atm} = 2\,\text{atm}$

Because the gas is *very slowly compressed*, we can assume that the process is isothermal. Work for an isothermal process is given by
$$W = nRT \ln(V_f/V_i)$$

T is constant throughout the entire process, and we can substitute nRT for some PV during the process. It is convenient to choose the PV from the initial state.

$$PV = P_0 V_0 = 2\,\text{atm}(101.3 \cdot 10^3\,\text{Pa/atm})(1.0\,\text{m}^3) = 202.6 \cdot 10^3\,\text{J}$$
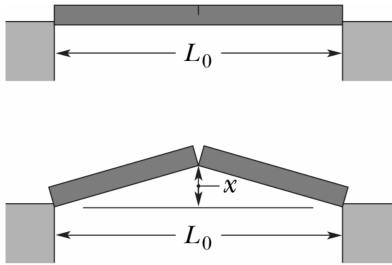
We obtain
$$W = P_0 V_0 \ln(0.4\,\text{m}^3/1.0\,\text{m}^3) = -1.86 \cdot 10^5\,\text{J}$$

However, this is the work done by the gas. The question wants the work done by the external force, so the final answer is
$$W_{ext} = 1.86 \cdot 10^5\,\text{J}$$

(∗)   3. Due to a temperature rise of $32°C$, a bar with a crack at its centre buckles upward. If the fixed distance L_0 is 3.77 m and the coefficient of linear expansion of the bar is $25 \cdot 10^{-6}/° \, C$, find the rise x of the centre.

Solution:
We are given: $\Delta T = 32 \, °C$, $\alpha = 25 \cdot 10^{-6}/°C$, and $L_0 = 3.77 \, m$.

If we let $L' = L_0 + \Delta L$ be the total length of the bar after expansion, then by $\Delta L = L_0 \alpha \Delta T$, we know $L' = 3.773 \, m$.

From the Pythagorean Theorem, we obtain $(L'/2)^2 - (L_0/2)^2 = x^2$.

$$3.5589 - 3.5532 = 0.00568 = x^2$$

This gives

$$x = 0.0754 \, m = 7.54 \, cm$$

(∗) 4. The temperature of a Pyrex disk is changed from $10.0°C$ to $60.0°C$. Its initial radius is $8.00\,\text{cm}$, and its initial thickness is $0.50\,\text{cm}$. Young's Modulus of pyrex is $67\,\text{GPa}$, and the linear thermal expansion coefficient of Pyrex is $3.2 \cdot 10^{-6}/°C$.

  (a) What is the change in volume of the disk?

  (b) If the disk is constrained rigidly across its faces before it is heated, what will be the stress in the disk after it is heated?

Solution:

(a) First, notice what is needed is the volumetric thermal expansion coefficient β, and only the linear thermal expansion coefficient α is given. Since $\beta = 3\alpha$, we can use the formula

$$\Delta V = \beta \Delta T V_0 = 3\alpha V_0 \Delta T$$

Where V_0 is the initial volume, ΔV is the change in volume, and ΔT is the change in temperature.

$$V_0 = 0.08^2 \pi (0.005) = 1.005 \cdot 10^{-4}\,\text{m}^3$$

$$\Delta T = 60 - 10 = 50\,\text{K}$$

Thus
$$\Delta V = 4.83 \cdot 10^{-8}\,\text{m}^3$$

(b) The initial thickness is $h = 0.005\,\text{m}$. When the disk is heated by $50°C$, its unrestrained height would be $h' = 0.0050008\,\text{m}$ tall. However, we have constrained it: so $\Delta h = 8 \cdot 10^{-7}\,\text{m}$, and $\Delta h/h = 1.6 \cdot 10^{-4}$. From the Young's modulus formula, we have

$$\frac{F}{A} = Y\frac{\Delta h}{h} = 67 \cdot 10^9 (1.6 \cdot 10^{-4}) = 1.07 \cdot 10^7\,\text{Pa}$$

(∗)   5. The giant hornet *Vespa mandarinia japonica* preys on Japanese bees. However, if a hornet attempts to invade a beehive, several hundred of the bees quickly form a compact ball around the hornet to stop it. They overheat the hornet by quickly raising their body temperatures from the normal 35°C to 47°C, which is lethal to the hornet but not to the bees. Assume the following: 500 bees form a ball of radius $R = 2.0$ cm for a time $t = 20$ min, the primary loss of energy by the ball is by thermal radiation, the ball's surface has emissivity $\epsilon = 0.80$, and the ball has uniform temperature. On average, how much additional energy must each bee produce during the 20 minutes to maintain 47°C?

Solution:

- The normal body temperature of the bee is

$$T_s = 35 + 273 = 308 \, \text{K}$$

- The new temperature of the bee sphere is

$$T_b = 47 + 273 = 320 \, \text{K}$$

- The surface area of the bee sphere is

$$A = (4\pi)0.02^2 = 0.00503 \, \text{m}^2$$

The net rate of additional radiation from the bee sphere will be

$$H_{net} = \sigma \epsilon A(T_b^4 - T_s^4) = 0.34 \, \text{W}$$

where σ is the Stefan Boltzmann constant. The total heat transfer from the ball in 20 minutes will be

$$Q = (0.34 \, \text{W})(20 \, \text{min})(60 \, \text{sec/min}) = 407 \, \text{J}$$

Since there were 500 bees contributing, each bee, on average, produced

$$\frac{407}{500} = 0.81 \, \text{J}$$

of additional energy.

($\ast\ast$)   6. Suppose that when Pluto is at its average distance of 39.5 AU from the Sun, it has an average temperature of $-235°$C. Compute the albedo α_p of Pluto. Note that 1 AU = 1 Astronomical Unit = distance from Sun to Earth. The solar constant at Earth is $1367\,\text{W/m}^2$.

Solution: We must first calculate the solar constant at Pluto. Since the solar constant at Earth is $1367\,\text{W/m}^2$, and Pluto is 39.5 times farther away from the Sun than Earth is, the solar constant at Pluto is given by

$$S_p = \frac{1367}{39.5^2} = 0.876\,\text{W/m}^2$$

Let r_p denote the radius of Pluto, and let α_p be the albedo of Pluto.

The net rate of heat in will therefore be

$$H_{in} = (1 - \alpha_p)S_p r_p^2 \pi$$

and the net rate of heat out will be

$$H_{out} = \epsilon \sigma 4\pi r_p^2 T^4$$

Approximate the emissivity of Pluto by $\epsilon \approx 1$

At equilibrium,

$$H_{in} = H_{out}$$

so

$$(1 - \alpha_p)S_p r_p^2 \pi = \sigma 4\pi r_p^2 T^4$$

Thus

$$1 - \alpha_p = 4\sigma(-235 + 273)^4/S_p = 0.54$$

So

$$\alpha_p = 0.46$$

(∗∗)　7. On a hot day, a cold beverage can is placed on an insulating surface and covered with a wet towel, such that there is no heat exchange between the surface and the bottom of the can. Assume that the only energy exchanges taking place are evaporation and radiation through the top and side surfaces. The towel and beverage have temperature $T = 15°C$, the environment has temperature $T_{env} = 32°C$, and the towel/beverage package is a cylinder with radius $r = 2.2\,\text{cm}$ and height $h = 10\,\text{cm}$. Approximate the emissivity as $\epsilon = 1$, and neglect other energy changes. At what rate $\Delta m/\Delta t$ is the container losing water mass? The latent heat of vaporization of water is $L = 2260\,\text{J/g}$.
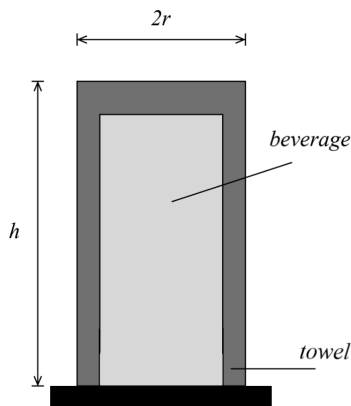
Solution:
The total area in consideration will be the area of the top and sides, which is given by

$$A = \pi r^2 + 2\pi r h = 0.0153\,\text{m}^2$$

Then we convert the temperature measure to Kelvin. With $T_{env} = 305\,\text{K}$ and $T = 288\,\text{K}$, we will be able to solve the following equation:

Since we are to assume that the rate of energy lost to evaporation is the same as the net energy gained via the radiation exchange, we have

$$L\frac{\Delta m}{\Delta t} = \sigma \epsilon A (T_{env}^4 - T^4)$$

where the left side is the latent heat of vaporization and the right size is the rate of energy gained from radiation.
Solving for the rate of evaporation, we obtain

$$\frac{\Delta m}{\Delta t} = 6.83 \cdot 10^{-4}\,\text{g/s}$$

(∗∗) 8. Two containers of equal volume, $V_1 = V_2 = V$, are connected by a small tube with a valve, as shown. Initially, the valve is closed and the two volumes contain a monatomic gas at pressures P_1 and P_2 and temperatures T_1 and T_2, respectively. After the valve is opened, what will be

　(a) the final pressure P_f and

　(b) the final temperature T_f inside the joint volume?

Neglect heat lost from the system. Express your answers in terms of P_1, P_2, T_1, T_2.
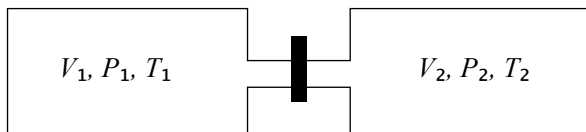
Solution:

The work for an adiabatic process is

$$W = nC_V(T_1 - T_2)$$

This provides an expression for the work done by the gas initially in container one:

$$W_1 = n_1 C_V(T_1 - T_f)$$

The work done by the gas initially in container two is

$$W_2 = n_2 C_V(T_2 - T_f)$$

The net work done by all gases must be zero because there was no external work by any force (conservation of energy).

$$W_1 + W_2 = 0$$

$$n_1 C_V(T_1 - T_f) + n_2 C_V(T_2 - T_f) = C_V(n_1 T_1 + n_2 T_2) - C_V T_f(n_1 + n_2) = 0$$

Thus,

$$T_f = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \tag{8.1}$$

From the ideal gas law, we have

$$
\begin{aligned}
P_1 V &= n_1 R T_1 & (8.2) \\
P_2 V &= n_2 R T_2 & (8.3) \\
P_f(2V) &= (n_1 + n_2) R T_f & (8.4)
\end{aligned}
$$

Adding (8.2) and (8.3), we obtain

$$(P_1 + P_2)V = R(n_1 T_1 + n_2 T_2) \tag{8.5}$$

Dividing (8.5) by (8.4), we obtain

$$\frac{(P_1 + P_2)V}{2P_f V} = \frac{R(n_1 T_1 + n_2 T_2)}{R T_f(n_1 + n_2)} \tag{8.6}$$

(a) From (8.6), we see that the right hand side is 1 because of (8.1), we obtain

$$P_f = \frac{P_1 + P_2}{2}$$

(b) For T_f, we can substitute the following two equations into (8.1):
Substituting

$$n_1 = \frac{P_1 V}{R T_1}$$

$$n_2 = \frac{P_2 V}{R T_2}$$

into (8.1) we obtain

$$T_f = \frac{T_1 T_2 (P_1 + P_2)}{P_1 T_2 + P_2 T_1}$$

(∗∗)   9. Two $50\,$g ice cubes are dropped into $200\,$g of water in a thermally insulated container. The water is initially at $25°C$, and the ice comes directly from a freezer at $-15°C$.

   (a) What is the final temperature of the mixture at thermal equilibrium?

   (b) What is the final temperature if only one ice cube is used?

The latent heat of fusion of water is $L_w = 336$ J/g. The specific heat capacity of water is $c_w = 4186$J/kg°C. The specific heat capacity of ice is $c_i = 2030$ J/kg°C.

Solution:

(a) Two Ice cubes:

- First, we calculate the amount of heat required to cool the water to 0°C.

$$Q = (0.2)(4186)(0 - 25) = -20930\,\text{J}$$

- Next, we calculate the amount of heat required to bring all of the ice to 0°C.

$$Q = 2(0.05)(2030)(0 - (-15)) = 3045\,\text{J}$$

- Finally, we calculate the amount of energy it would take to melt all of the ice.

$$Q = 2(50)(336) = 33600\,\text{J}$$

We can see that
$$3045 + 33600 = 36645 > 20930$$
, so then we must conclude that the heat energy that the water loses to the ice will bring all of the ice to 0°C, but will not be able to melt it all. Thus we would be left with a mixture of water and ice at a temperature of 0°C.

(b) One Ice cube:

- The amount of heat energy to remove from the water to cool it to 0°C is 20930 J.
- The amount of heat energy to bring one ice cube to 0°C is $3045/2 = 1522.5$ J
- The amount of heat energy required to melt one ice cube is $33600/2 = 16800$ J.

We see that
$$1522.5 + 16800 = 18322.5 < 20930$$

This means that the water will cool down substantially in heating up, then melting, the ice cube, but the water will be left at a positive equilibrium temperature.

Now we must solve for the final temperature T_f.

- The heat released by the water bath, as it cools from 25°C to its final temperature, is

$$Q_w = m_w c_w (T_f - 25)$$

- The heat absorbed by the ice as it warms, melts, then warms again is

$$Q_i = m_i c_w (T_f - 0) + 18322.5$$

Note that the 18322.5 is the energy required to bring the ice cube to $0°C$ and to melt it.

- The net heat transferred in/out of the system must be 0, so

$$Q_w + Q_i = m_w c_w (T_f - 25) + m_i c_w (T_f - 0) + 18322.5 = 0$$

With $m_w = 0.2\,\text{kg}$, $m_i = 0.05\,\text{kg}$, $c_w =$4186 J/kg$°C$, we solve for T_f and obtain $T_f = 2.5°C$.

(**) 10. An insect is caught at the midpoint of a spider-web thread. The thread breaks under a stress of $8.2 \cdot 10^8 \, \text{N/m}^2$, and a strain of 2.00. Initially, it was horizontal and had a length of $2.00 \, \text{cm}$ and a cross-sectional area of $8.00 \cdot 10^{-12} \, \text{m}^2$. Assume that as the thread is stretched, its volume remains constant, and its cross-sectional area decreases uniformly along the thread. If the weight of the insect puts the thread on the verge of breaking, what is the insect's mass?
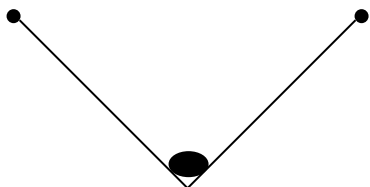
Solution:
Let $A = 8.00 \cdot 10^{-12} \, \text{m}^2$ be the initial cross-sectional area of the thread, and $l = 0.02 \, \text{m}$ be the initial total length.

First we want to calculate Young's modulus for this material. From Hooke's law,

$$\frac{F}{A'} = Y \frac{\Delta l}{l} \tag{10.1}$$

we can plug in stress $F/A' = 8.2 \cdot 10^8 \, \text{Pa}$ and strain $\Delta l/l = 2$. This gives Young's modulus as $Y = 4.1 \cdot 10^8 \, \text{Pa}$.

Now we calculate the strain in the string after the insect has landed at its midpoint. Let l' be the length of the thread after stretching, thus $\Delta l = l' - l$. We have

$$\frac{l' - l}{l} = \Delta l/l = 2$$

This yields

$$l' = 3l = 3(0.02) = 0.06 \, \text{m}$$

Now we use the length of the stretched string to find the new cross-sectional area. Let A' be the cross sectional area of the string after stretching. Since the volume remains constant, we have

$$V = 8 \cdot 10^{-12}(0.02) = A' \cdot 0.06$$

This gives $A' = 2.67 \cdot 10^{-12} \, \text{m}^2$.

Since the maximum stress $F/A' = 8.2 \cdot 10^8$, this gives $F = 0.00219 \, \text{N}$. Note that this $F = T$ is the tension in the string.

Since this system is in equilibrium and the force from the string pulls along the direction of the string, we have

$$2T \sin \theta = mg \tag{10.2}$$

for the vertical direction. Since we know the horizontal distance of the fly from an endpoint, we can use the Pythagorean Theorem to find the vertical distance $h = 0.0283$. Thus

$$\sin \theta = 0.0283/0.03 = 0.943$$

Finally, solving for m in (10.2), we get $m = 0.421 \, \text{grams}$.

(**) 11. Shown below is a 300 kg cylinder that is horizontal. Three steel wires support the cylinder from a ceiling. Wires 1 and 3 are attached at the ends of the cylinder, and wire 2 is attached at the centre. Initially, (before the cylinder was put in place) wires 1 and 3 were 2.000 m long and wire 2 was 6 mm longer than that. Now, (with the cylinder in place) all three wires have been stretched to a cross sectional area of $2.00 \cdot 10^{-6} \, \text{m}^2$. Young's Modulus for steel is $Y = 200 \cdot 10^9 \, \text{N/m}^2$. What is the tension in,

(a) wire 1?

(b) wire 2?
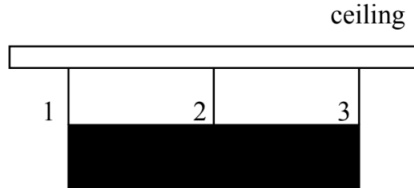
Solution:

Let T_1 be the tension in wire 1, and T_2 be the tension in wire 2. The tensions in wires 1 and 3 are equal, by symmetry.

Also, let A_1, A_2 be the initial cross-sectional areas of the wires, let $d = 0.006 \, \text{m}$ and $l_1 + d = l_2$.

We will denote the lengths and cross-sectional areas of the wires after the cylinder is placed with primes. Thus $l_1' = l_2'$, and $\Delta l_1 = l_1' - l_1$, and $\Delta l_2 = l_2' - l_2$. For cross-sectional areas, $A_1' = A_2' = 2 \cdot 10^{-6} \, \text{m}^2$.

Equating $\Delta l_1 = l_1' - l_1$ and $\Delta l_2 = l_2' - l_2$ while substituting in $l_1 + d = l_2$, we obtain

$$\Delta l_1 = \Delta l_2 + d \tag{11.1}$$

Physically this corresponds to how l_1 needs to stretch $0.006 \, \text{m}$ more to reach the same length. From equilibrium, we have $2T_1 + T_2 = 300 \cdot g$, and from the constant volume equation we have $A_2' l_2' = A_2 l_2$ and $A_1' l_1' = A_1 l_1$.

From Hooke's Law

$$\frac{T_1}{A_1'} = Y \frac{\Delta l_1}{l_1} \tag{11.2}$$

$$\frac{T_2}{A_2'} = Y \frac{\Delta l_2}{l_2} \tag{11.3}$$

we rearrange to obtain

$$\Delta l_1 = \frac{l_1 T_1}{A_1' Y} \tag{11.4}$$

$$\Delta l_2 = \frac{l_2 T_2}{A_2' Y} \tag{11.5}$$

Substituting (11.4) and (11.5) into (11.1), we obtain

$$\frac{l_1 T_1}{A_1' Y} = \frac{l_2 T_2}{A_2' Y} + d \tag{11.6}$$

$$= \frac{l_2 T_2 + Y d A_2'}{A_2' Y} \tag{11.7}$$

Rearranging (11.7),
$$\frac{A_2' l_1 T_1}{A_1'} = l_2 T_2 + Y d A_2' = l_1 T_1 \tag{11.8}$$

$$T_1 \frac{l_1}{l_2} = T_2 + \frac{Y A_2' d}{l_2} \tag{11.9}$$

From the equilibrium condition,
$$2T_1 + T_2 = 300 \cdot g \tag{11.10}$$

Using (11.9) and (11.10) we get a linear system:

$$\begin{cases} 0.997 T_1 - T_2 = 1196.4 \\ 2T_1 + T_2 = 2940 \end{cases}$$

(a) Adding the two equations,
$$2.997 T_1 = 4136.4$$

$$T_1 = 1380 \,\mathrm{N}$$

(b) Back-substituting,
$$T_2 = 180 \,\mathrm{N}$$

($**$) 12. A 50 litre tank is connected to a 15 litre tank through a short tube containing a pressure release valve that will only allow gas to pass from the larger tank to the smaller tank if the pressure in the larger tank exceeds the pressure in the smaller tank by $117,300$Pa. If at $17°$C the larger tank contains gas at atmospheric pressure and the smaller tank is evacuated, what will be the pressure P in the smaller tank when both tanks are at $162°$C?

Solution:
Let P_1 be the pressure in the larger tank at $162°$C, and $P_2 = P$ be the pressure in the smaller tank at $162°$C. Since both tanks end up at the same temperature,

$$T_1 = T_2 = 162°\text{C} = 435\,\text{K}$$

is the common temperature to the two tanks.

Let n_1 be the number of moles in the larger tank at $162°$C, and n_2 be the number of moles in the smaller tank at $162°$C.

The volumes of the two tanks are:
$$V_1 = 50\,\text{L} = 0.05\,\text{m}^3$$
$$V_2 = 15\,\text{L} = 0.015\,\text{m}^3$$

If the smaller tank is evacuated, it will have a pressure significantly lower than the pressure in larger tank (at $162°$C). Thus we can assume that air will flow across the valve, and at $162°$C,

$$P_1 = P_2 + 117300 \tag{12.1}$$

The total number of moles in the system can be calculated by using the initial amount of moles in the larger tank (the smaller tank has no moles in it after being evacuated).

$$
\begin{aligned}
n_{total} &= \frac{P_0 V_0}{R T_0} \tag{12.2}\\
&= \frac{(101.3 \cdot 10^3)(0.05)}{(8.31)(290)} \tag{12.3}\\
&= 2.1\,\text{mol} \tag{12.4}
\end{aligned}
$$

Thus we have
$$n_1 + n_2 = 2.1 \tag{12.5}$$

From the ideal gas law, we have $P_1 V_1 = n_1 R T_1$, and $P_2 V_2 = n_2 R T_2$. Plugging in the known variables into these two equations, we obtain
$$P_1 = 72297 n_1 \tag{12.6}$$
$$P_2 = 240990 n_2 \tag{12.7}$$

We now solve the system of equations:
$$
\begin{cases}
P_1 = P_2 + 117300\\
n_1 + n_2 = 2.1\\
P_1 = 72297 n_1\\
P_2 = 240990 n_2
\end{cases}
$$

$$
\begin{aligned}
P_1 &= 72297 n_1\\
&= 72297(2.1 - n_2)\\
&= 151824 - 72297 n_2
\end{aligned}
$$

Since $n_2 = P_2/240990$, we have
$$P_1 = 151824 - 0.3P_2 \tag{12.8}$$

Rearranging (12.8), $P_1 + 0.3P_2 = 151824$. Coupling this with (12.1), $P_1 = P_2 + 117300$, we can solve the linear system.

$$P_2 + 117300 + 0.3P_2 = 151824$$
$$1.3P_2 = 34524$$
$$P_2 = 26557\,\text{Pa} = P$$

Thus $P = 26557\,\text{Pa} =$ the pressure in the smaller tank at $162°\text{C}$.

(**) 13. A 20.0 g copper ring at $0.0°C$ has an inner diameter of $D = 2.54000$ cm. An aluminum sphere at $100.0°C$ has a diameter of $d = 2.54508$ cm. The sphere is put on top of the ring, and the two are allowed to come to thermal equilibrium, with no heat lost to the surroundings. The sphere just passes through the ring at the equilibrium temperature. What is the mass of the sphere?

The coefficient of volume expansion of aluminum is $\beta_{Al} = 7.2 \cdot 10^{-5}/°C$, and the coefficient of volume expansion of copper is $\beta_{Cu} = 5.1 \cdot 10^{-5}/°C$. The specific heat capacity of aluminum is $c_{Al} = 910$ J/kg$°C$, and the specific heat capacity of copper is $c_{Cu} = 390$ J/kg$°C$.
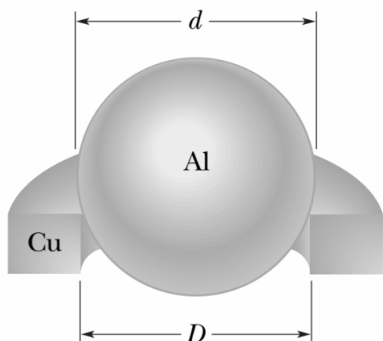
Solution: First, we need the coefficients of linear expansion of each material. We know that in general for a material, $\beta = 3\alpha$.

Thus, $\alpha_{Al} = 2.4 \cdot 10^{-5}/°C$ and $\alpha_{Cu} = 1.7 \cdot 10^{-5}/°C$.

Let L'_{Al} be the diameter of the aluminum sphere after it has come to equilibrium temperature. Thus

$$\begin{aligned} L'_{Al} &= d + \Delta L_{Al} \\ &= d(1 + \alpha_{Al}\Delta T) \\ &= d(1 + \alpha_{Al}(T_f - 100)) \end{aligned}$$

Let L'_{Cu} be the inner diameter of the copper ring after it has come to equilibrium temperature. Thus

$$\begin{aligned} L'_{Cu} &= D + \Delta L_{Cu} \\ &= D(1 + \alpha_{Cu}\Delta T) \\ &= D(1 + \alpha_{Cu}(T_f - 0)) \end{aligned}$$

We know that, at equilibrium temperature, the diameters are the same. Thus $L'_{Al} = L'_{Cu}$

$$d(1 + \alpha_{Al}(T_f - 100)) = D(1 + \alpha_{Cu}(T_f))$$

Solving for T_f, we obtain

$$T_f = \frac{D - d + 100d\alpha_{Al}}{d\alpha_{Al} - D\alpha_{Cu}} = 57.43°C$$

Now that we have the equilibrium temperature, we can use the fact that no heat was lost to the environment to set up the following equation:

The heat lost by the aluminum sphere is given by

$$Q_{Al} = m_{Al}c_{Al}(T_f - 100)$$

The heat gained by the copper ring is given by

$$Q_{\mathrm{Cu}} = m_{\mathrm{Cu}} c_{\mathrm{Cu}} (T_f - 0)$$

Since no heat was lost to the environment, $Q_{\mathrm{Al}} + Q_{\mathrm{Cu}} = 0$

This yields

$$m_{\mathrm{Al}} c_{\mathrm{Al}} (T_f - 100) + m_{\mathrm{Cu}} c_{\mathrm{Cu}} (T_f - 0) = 0$$

With T_f known, we solve for

$$m_{\mathrm{Al}} = -\frac{m_{\mathrm{Cu}} c_{\mathrm{Cu}} T_f}{c_{\mathrm{Al}} (T_f - 100)} = 11.56 \cdot 10^{-3} \, \mathrm{kg}$$

(∗ ∗ ∗) 14. A sample of gas undergoes a transition from an initial state a to a final state b by three different paths, as shown in the P-V diagram, where $V_b = 5.00V_i$. The energy transferred to the gas as heat in process 1 is $10P_iV_i$.

(a) How many degrees of freedom does the sample of gas have?

(b) Find the energy transferred to the gas as heat in process 2.

(c) Find the change in internal energy that the gas undergoes in process 3.

Express your answers in terms of P_i, V_i.
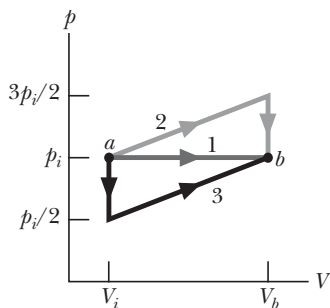
Solution:

(a) We will first try and get more data about path 1. Since it is an isobaric process because pressure is constant, the work done will be

$$W = P_i(V_b - V_i) = 4P_iV_i$$

From the ideal gas law $V_i/T_i = V_b/T_b$, we can find that

$$T_b = 5T_i$$

$$\Delta T = 4T_i$$

The change in internal energy will be

$$\begin{aligned} \Delta U &= nC_v\Delta T \\ &= nC_V(4T_i) \\ &= 4C_V(nT_i) \\ &= 4C_V(P_iV_i/R) \end{aligned}$$

Then we can use the first law of thermodynamics to find a relationship between C_V and R.

$$\Delta U = Q - W$$

$$4C_V P_iV_i/R = 10P_iV_i - 4P_iV_i = 6P_iV_i$$

$$C_V = 3R/2$$

Thus we conclude that it is a monatomic gas and has 3 degrees of freedom. Note that the three degrees of freedom are simply movement in the x, y, and z directions.

(b) The work for the first segment of path 2 can be found using the area under the line. We use the formula for the area of a trapezoid.

$$W = (V_b - V_i)(P_i + 3P_i/2)/2 = 5P_iV_i \tag{14.1}$$

The change in temperature in the first segment of path 2 can be found using the ideal gas law. We have

$$P_iV_i/T_i = P_bV_b/T_b(5V_i)(3P_i/2)/(T)$$

$$T = 15T_i/2$$

This means that $\Delta T = 13/2T_i$.

The change in internal energy will be

$$
\begin{aligned}
\Delta U &= nC_V(13/2T_i) & (14.2)\\
&= (13/2)C_VP_iV_i/R & (14.3)\\
&= (13/2)(3/2)P_iV_i & (14.4)\\
&= 39P_iV_i/4 & (14.5)
\end{aligned}
$$

We can use the first law of thermodynamics to find that $\Delta Q = \Delta U + W$ for this first segment of the path (using (14.1) and (14.5)) is

$$39P_iV_i/4 + 5P_iV_i = 59P_iV_i/4 \tag{14.6}$$

For the second (vertical) segment of the path, there is no work done. Thus the heat added (at constant volume) will depend only on the temperature difference. We know that the temperature at b is $5T_i$, so then the change in heat energy for this second segment will be

$$Q = nC_V(-5/2T_i) = -15P_iV_i/4 \tag{14.7}$$

Adding these two heats, we find that the total change in heat energy over path 2 is

$$\Delta Q = 59P_iV_i/4 - 15P_iV_i/4 = 11P_iV_i$$

(c) Internal energy is independent of path, so then

$$\Delta U = nC_V(4Ti) = 6P_iV_i$$

(∗ ∗ ∗) 15. Two rods are perfectly constrained between rigid constraints, as shown. Initially, at $15°C$, they have no stress in them. Rod A has a cross sectional area of $40\,\text{cm}^2$, and rod B has a cross sectional area of $50\,\text{cm}^2$. Rod A is $30\,\text{cm}$ long, and rod B is $20\,\text{cm}$ long. Rod A has Young's modulus of $10^{10}\,\text{Pa}$, and rod B has Young's modulus of $2 \cdot 10^9\,\text{Pa}$. Rod A has thermal expansion coefficient of $2.4 \cdot 10^{-5}/°\text{C}$, and rod B has thermal expansion coefficient of $5.7 \cdot 10^{-6}/°\text{C}$. If the temperature of both is raised to $40°C$,

     (a) What will be the stress in rod A?

     (b) What will be the stress in rod B?

     (c) What will be the length of rod A?

     (d) What will be the length of rod B?
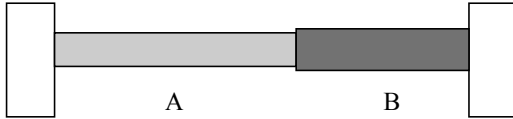
Solution: Assume that both rods are under compression. The forces F_A and F_B at the interface between the rods are in opposite directions, and are of equal magnitude, so $F_A = -F_B$. If both rods are under compression, F_A is pointing in the leftwards direction, and F_B is pointing in the rightwards direction. In each rod, there will be elongation/compression due to two sources:

- heating

- compressive forces

Consider the change in length δ_A of rod A. There will be *negative* contribution due to F_A, and a *positive* contribution due to the thermal expansion. Since F_A is already negative (pointing in the leftward direction) we do not need to introduce an additional negative sign in front of it.

$$\delta_A = \alpha_A L_A \Delta T + \frac{F_A L_A}{A_A Y_A} \tag{15.1}$$

Consider the change in length δ_B of rod B. There will be *negative* contribution due to F_B, and a *positive* contribution due to the thermal expansion. Since F_B is positive (pointing in the rightward direction), we need to introduce a negative sign in front of it.

$$\delta_B = \alpha_B L_B \Delta T - \frac{F_B L_B}{A_B Y_B} \tag{15.2}$$

Since the rods are constrained between rigid constraints, the total combined length of the rods will not change. We have

$$\delta_A = -\delta_B \tag{15.3}$$

Combining equations (15.1) and (15.2) via (15.3), we have

$$\alpha_A L_A \Delta T + \frac{F_A L_A}{A_A Y_A} = -\left(\alpha_B L_B \Delta T - \frac{F_B L_B}{A_B Y_B} \right) \tag{15.4}$$

Plugging in $F_A = -F_B$, and rearranging,

$$F_A \left(\frac{L_A}{A_A Y_A} + \frac{L_B}{A_B Y_B} \right) = -\Delta T (\alpha_A L_A + \alpha_B L_B) \tag{15.5}$$

Isolating F_A:

$$F_A = \frac{-\Delta T (\alpha_A L_A + \alpha_B L_B)}{\frac{L_A}{A_A Y_A} + \frac{L_B}{A_B Y_B}} \tag{15.6}$$

Plugging in the values gives

$$F_A = \frac{-25(2.4 \cdot 10^{-5} \cdot 0.3 + 5.7 \cdot 10^{-6} \cdot 0.2)}{\frac{0.3}{0.004 \cdot 10^{10}} + \frac{0.2}{0.005 \cdot 2 \cdot 10^9}} = -7581 \, \text{N}$$

Since the force F_A came out negative, our initial assumption that both rods were under compressive stress is correct.

(a) The (compressive) stress in rod A can then be calculated as

$$\frac{7581}{0.004} = 1.89 \cdot 10^6 \, \text{Pa}$$

(b) The (compressive) stress in rod B is

$$\frac{7581}{0.005} = 1.52 \cdot 10^6 \, \text{Pa}$$

(c) Plugging the stresses back into the formula for δ_A gives

$$\delta_A = 2.4 \cdot 10^{-5} \cdot 0.3 \cdot 25 - \frac{7581 \cdot 0.3}{0.004 \cdot 10^{10}} = 1.23 \cdot 10^{-4} \, \text{m}$$

Then the final length of rod A is

$$L_A + \delta_A = 0.300123 \, \text{m}$$

(d) Since $\delta_B = -\delta_A$, we can calculate

$$\delta_B = -1.23 \cdot 10^{-4} \, \text{m}$$

Then the final length of rod B is

$$L_B + \delta_B = 0.199877 \, \text{m}$$

(∗ ∗ ∗) 16. The figure shows the cross section of a wall made of three layers. The layer thicknesses are L_1, $L_2 = 0.700L_1$, and $L_3 = 0.350L_1$. The thermal conductivities are k_1, $k_2 = 0.900k_1$, and $k_3 = 0.800k_1$. The temperatures at the left and right sides of the wall are $T_H = 30.0°\text{C}$ and $T_C = -15.0°\text{C}$. Thermal conduction is steady.

(a) What are the temperatures at each interface? (where two layers meet)

(b) If k_2 were, instead, equal to $1.1k_1$, would the rate at which energy is conducted through the wall be greater than, less than, or the same as previously?

(c) Using the k values from part (b), what would be the values of the temperatures at the two interfaces?

(d) Using the k values from part (a), if the thermal conductivity of layer 1 is $k_1 = 50\,\text{W/m} \cdot \text{K}$, and the thickness of layer 1 is $L_1 = 60\,\text{cm}$, what are the R values of each of the walls?
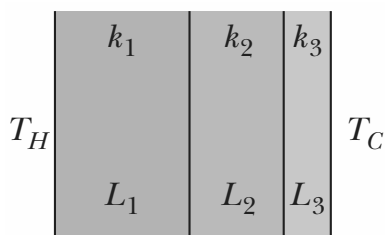
Solution:

(a) We know that the rate of heat transfer across each layer must be equal to each other and to the rate across the entire wall. Thus $H_1 = H_2 = H_3$.

Let A be the cross sectional area common to the three walls, and let T_{12} be the temperature at the interface between walls 1 and 2, and let T_{23} be the temperature at the interface between walls 2 and 3.

We have

$$H_1 = \frac{Ak_1(T_H - T_{12})}{L_1}$$

$$H_2 = \frac{Ak_2(T_{12} - T_{23})}{L_2}$$

$$H_3 = \frac{Ak_3(T_{23} - T_C)}{L_3}$$

Since it is a steady thermal conduction,

$$H_1 = H_2 = H_3$$

Substituting in our equations,

$$\frac{Ak_1(T_H - T_{12})}{L_1} = \frac{Ak_2(T_{12} - T_{23})}{L_2} = \frac{Ak_3(T_{23} - T_C)}{L_3}$$

We can do some cancellations to obtain

$$\frac{k_1(T_H - T_{12})}{L_1} = \frac{0.9k_1(T_{12} - T_{23})}{0.7L_1} = \frac{0.8k_1(T_{23} - T_C)}{0.35L_1}$$

$$\frac{(T_H - T_{12})}{1} = \frac{9(T_{12} - T_{23})}{7} = \frac{16(T_{23} - T_C)}{7}$$

This provides us with a system of equations:

$$\begin{cases} 16T_{12} - 9T_{23} = 210 \\ 25T_{23} - 9T_{12} = -240 \end{cases}$$

Solving yields $T_{12} = 9.7°$C, and $T_{23} = -6.1°$C.

(b) A greater heat conductivity constant would *increase* the rate of conductive heat transfer.

(c) From
$$H_1 = H_2 = H_3$$

Substituting in new values,

$$\frac{Ak_1(T_H - T_{12})}{L_1} = \frac{Ak_2(T_{12} - T_{23})}{L_2} = \frac{Ak_3(T_{23} - T_C)}{L_3}$$

We would obtain

$$\frac{k_1(T_H - T_{12})}{L_1} = \frac{1.1k_1(T_{12} - T_{23})}{0.7L_1} = \frac{0.8k_1(T_{23} - T_C)}{0.35L_1}$$

and thus
$$7(T_H - T_{12}) = 11(T_{12} - T_{23}) = 16(T_{23} - T_C)$$

Solving the system of equations we obtain $T_{12} = 8.3°$C, and $T_{23} = -5.5°$C

(d) The R value (thermal resistance) of each wall is given by $R = L/k$. Thus we can calculate

$$R_1 = L_1/k_1 = 0.012 \ \text{m}^2 \, \text{K/W}$$
$$R_2 = L_2/k_2 = 9.3 \cdot 10^{-3} \ \text{m}^2 \, \text{K/W}$$
$$R_3 = L_3/k_3 = 5.25 \cdot 10^{-3} \ \text{m}^2 \, \text{K/W}$$

Useful Constants and Conversion Ratios:
R = Ideal Gas constant = $8.31451 \, \text{J/molK}$,   1 atm = $1.013 \times 10^5 \, \text{Pa}$,   1 atm · litre = $101.3 \, \text{J}$
σ = Stefan-Boltzmann constant = $5.6704 \times 10^{-8} \, \text{W/m}^2\text{K}^4$, $\gamma_{\text{air}} = 1.4$, $C_{V_{\text{air}}} = 20.8 \, \text{J/molK}$
ρ_{water} = Density of water = $1 \, \text{gram/cm}^3 = 1000 \, \text{kg/m}^3$

Mechanics:
Linear Motion: $x = x_0 + \frac{1}{2}(v_0 + v)t$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2a(x - x_0)$

Circular Motion: $a_c = \dfrac{v^2}{r}$

Forces: $\mathbf{F} = m\mathbf{a} = \dfrac{d}{dt}\mathbf{p}$,   Friction: $|\mathbf{F}| = \mu|\mathbf{N}|$,   Spring: $\mathbf{F} = -k\mathbf{x}$,   Damping: $\mathbf{F} = -b\mathbf{v}$
Buoyant $|\mathbf{F}| = \rho V g$

$W = \text{Work} = \displaystyle\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r} = \mathbf{F} \cdot \Delta\mathbf{r}$, $K = \frac{1}{2}mv^2$, $\Delta U_{\text{gravity}} = mg\Delta h$, $\Delta U_{\text{spring}} = \frac{1}{2}kx^2$

$P = \dfrac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$

Thermodynamics:

Thermal Expansion: $\Delta L = \alpha L_0 \Delta T$,   Stress and Strain: $\dfrac{|\mathbf{F}|}{A} = Y\dfrac{\Delta L}{L}$,   Ideal Gas Law: $PV = nRT$

$K_{\text{av}} = \frac{3}{2}kT$

Thermal Conductivity: $I = \dfrac{\Delta Q}{\Delta t} = kA\dfrac{\Delta T}{\Delta x}$

Black Body Radiation: $P = e\sigma A(T_B^4 - T_S^4)$, $\lambda_{\text{max}} T = 2.8977685 \times 10^{-3} m \cdot K$
Solar Irradiation: $P = (1 - \alpha)S_c r^2 \pi$
Internal Energy: $U = nC_V T$
First Law of Thermodynamics: $dQ = dU + dW$ For an ideal gas, $dW = PdV$
Work for an *isothermal process* $W = nRT\ln(V_f/V_i)$
Work for an *adiabatic expansion* $TV^{\gamma-1} = $ constant, if the number of moles is constant $PV^\gamma = C$
where C is a constant and $\gamma = C_P/C_V$

Work for adiabatic process: $W = \displaystyle\int_{V_1}^{V_2} PdV = C\int_{V_1}^{V_2} \dfrac{dV}{V^\gamma} = \dfrac{C}{1 - \gamma}(V_2^{1-\gamma} - V_1^{1-\gamma})$

Heat Transfer: $Q = mc\Delta T$, $Q = mL$, $C_P = C_V + R$, $C_V = \dfrac{f}{2}R$, where $f = $ degrees of freedom.
$f = 3$ for monatomic and $f = 5$ for diatomic.
$dS = \dfrac{dQ}{T}$

$e = W/Q_H$, $COP_{\text{Cooling}} = \dfrac{|Q_C|}{|W|}$, $COP_{\text{Heating}} = \dfrac{|Q_H|}{|W|}$, $e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H}$

Integrals:
$$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C, n \neq 1 \qquad\qquad \int x^{-1} dx = \ln x + C$$

Trigonometry:
$$\sin\theta_1 + \sin\theta_2 = 2\cos\left(\dfrac{\theta_1 - \theta_2}{2}\right)\sin\left(\dfrac{\theta_1 + \theta_2}{2}\right)$$

Area and Volume:
Surface Area of a sphere: $A = 4\pi r^2$. Lateral surface area of a cylinder: $A = 2\pi rl$.
Area of a circle: $A = \pi r^2$. Volume of a cylinder: $V = l\pi r^2$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Oscillations:
$\omega = 2\pi f$, $T = \dfrac{1}{f}$, $x = A\cos(\omega t + \phi)$, $\omega^2 = \dfrac{k}{m}$

Damped Oscillations: $x = A_0 e^{-\frac{bt}{2m}}\cos(\omega t + \phi)$, where $\omega = \sqrt{w_0^2 - \left(\dfrac{b}{2m}\right)^2}$, $Q = 2\pi\dfrac{E}{\Delta E}$

Energy for damped $E = E_0 e^{-\frac{bt}{m}}$