Physics 157 Midterm 2 Review Package – **Solutions**

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as (∗) for easy, (∗∗) for medium, and (∗ ∗ ∗) for difficult. Difficulty is subjective, so do not be discouraged if you are stuck on a (∗) problem.

Solutions will be posted at: `https://ubcengineers.ca/tutoring/`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Want a warm up?	Short on study time?	Want a challenge?
These are the easier problems	These cover most of the material	These are some tougher questions
1, 2, 3	4, 7, 9	10, 11, 12

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Fundamentals of Physics / David Halliday, Robert Resnick, Jearl Walker. − 9th ed.
- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.
- A Student's Guide to Entropy / Don Lemons

All solutions prepared by the EUS.

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb wont help you understand the material.
- **Sleep**—Weve all been told we need 8 hours of sleep a night, university shouldnt change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!

(∗) 1. A constant current of 10 A flows through a resistor of 10 Ω which is kept at the constant temperature of 10°C.

(a) What is the rate of entropy change dS_R/dt of the resistor?

(b) What contribution dS_U/dt is made to the entropy change of the universe?

Solution:

(a) We know that entropy is defined by

$$dS_R = \frac{dQ}{T}.$$

If we're interested in time rate of change, we can take the derivative with respect to time which yields

$$\frac{dS_R}{dt} = \frac{1}{T} \cdot \frac{dQ}{dt}$$

because the temperature T is constant. Note that dQ/dt is the power dissipated by the resistor. Since $P = IV = I^2R$ (because of Ohm's law), we have

$$P = 10^2 \cdot 10 = 1000\,\text{W}$$

So then we might **wrongly** conclude that

$$\frac{dS_R}{dt} = \frac{1000}{273 + 10} = 3.53\,\text{J/sK}$$

This is **false** because, since the resistor is kept at a constant temperature, it will radiate that energy away. So then the net entropy change in the resistor is actually

$$\frac{dS_R}{dt} = 0$$

(b) The contribution to the universe will then be

$$\frac{dS_U}{dt} = 3.53\,\text{J/sK}.$$

(∗) 2. The Solar Constant at Earth's atmosphere is $1390\,\mathrm{W/m^2}$. The radius of the Sun is $695 \cdot 10^6$ m, and the average distance between the Earth and the Sun is $150 \cdot 10^9$ m. Find

(a) The temperature of the Sun (assuming it radiates as a black-body)

(b) The equilibrium temperature of Earth

Solution:

(a) We start by viewing a sphere centered at the sun and with a radius equal to the distance between earth and the sun (r_E). We know that the Solar Constant (in $\mathrm{W/m^2}$) is just the Sun's total power divided by the surface area of this sphere.

$$P = 1390 \cdot (4\pi r_E^2) = 3.93 \cdot 10^{26}\,\mathrm{W}$$

From the Stefan-Boltzmann Law: $P = A\sigma e T^4$. Using the radius of the Sun (r_S), and knowing that $e = 1$ for a black body, we get

$$P = (4\pi r_S^2)\sigma T_S^4$$

$$T_S = \left(\frac{3.93 \cdot 10^{26}}{(4\pi r_S^2)\sigma}\right)^{1/4} = 5810\,\mathrm{K}$$

Where σ is the Stephan-Boltzmann constant.

(b) The equilibrium temperature of Earth (T_E) occurs when the radiation absorbed from the Sun equals the radiation emitted by the Earth. For absorption, we consider the cross-sectional area of the Earth: a 2-dimensional disk. For emission, we use the entire surface area of the Earth. We have the power balance

$$(1-\alpha)\pi r_E^2(1390) = 4\pi r_E^2 \sigma e T_E^4$$

Here, let's assume that $e = 1$ and $\alpha = 0$ (full absorbtion). Simplifying this equation and solving for T_E gives us

$$T_E = \left(\frac{1390}{4\sigma}\right)^{1/4} = 280\,\mathrm{K}$$

(∗∗) 3. Pluto's diameter is approximately 2000 km and it is is 40 times farther away from the Sun than the Earth. The solar constant at the Earth's atmosphere is $1390\,\mathrm{W/m^2}$. Assume emissivity is 1. The albedo of Pluto is 0.4.

(a) What is the total power absorbed by Pluto?

(b) What is the temperature of Pluto?

(c) Assume that the atmospheric pressure is half that of Earth's. What is the density of the molecules on Pluto's surface? (Hint: use $R = 8.2 \cdot 10^{-5}\,\mathrm{m^3atm/k/mol}$)

Solution:

(a) We know that the intensity of the Sun's radiation at any distance is the total power of the Sun divided by the surface area of a sphere whose radius is equal to that distance. Applying this, we see

$$P_S = 1390(4\pi r_E^2) = I_P(4\pi r_P^2)$$

$$I_P = \frac{1390(4\pi r_E^2)}{4\pi r_P^2} = \frac{1390 r_E^2}{40^2 r_E^2} = 0.869\,\mathrm{W/m^2}$$

We use the equation $P_P = I_P \pi r_p^2(1 - a_P)$, where a_P is the albedo. We get

$$P_P = 0.869\pi(1 \cdot 10^6)^2(0.6) = 1.637 \cdot 10^{12}\,\mathrm{W}$$

(b) We know that the power in equals the power out:

$$P_P = P_{out} = 4\pi r_P^2 \sigma T_P^4$$

$$T_P = \left(\frac{I_P \pi r_p^2(1 - a_P)}{4\pi r_P^2 \sigma}\right)^{1/4} = \left(\frac{I_P(1 - a_P)}{4\sigma}\right)^{1/4} = 38.9K$$

(c) We know the density of molecules will just be the number of molecules divided by the volume they occupy. We can rearrange $PV = nRT$ to get

$$\frac{n}{V} = \frac{P}{RT}$$

We multiply both sides by Avogadro's number, $N_a = 6.02 \cdot 10^{23}$

$$\text{density} = \frac{nN_a}{V} = PN_aRT = \frac{0.5\,\mathrm{atm}(6.02 \cdot 10^{23})}{8.2 \cdot 10^{-5}(38.9\,\mathrm{K})} = 9.44 \cdot 10^{25}\,\mathrm{molecules/m^3}$$

(∗) 4. One mole of gas in a container is initially at a temperature 127°C. It is suddenly expanded to twice its initial volume without heat exchange with the outside. Then it is slowly compressed, holding the temperature constant, to the original volume. The final temperature is found to be −3° C.

(a) What is the coefficient γ of the gas?

(b) What change ΔS in entropy, if any, has occurred?

Solution:

(a) For any pair of T, V during an adiabatic process,

$$T_0 V_0^{\gamma-1} = TV^{\gamma-1}$$

So then since $T_0 = 400\,\mathrm{K}$ and $T = 270\,\mathrm{K}$, we have

$$400V_0^{\gamma-1} = 270(2V_0)^{\gamma-1}$$

Thus $400/270 = 2^{\gamma-1}$. Solving for γ, we obtain

$$\gamma = 1.57$$

(b) **Adiabatic Process**

During the adiabatic process, there is no change in entropy.

$$dS = \frac{dQ}{T}$$

and since $dQ = 0$, we conclude that $dS = 0$.

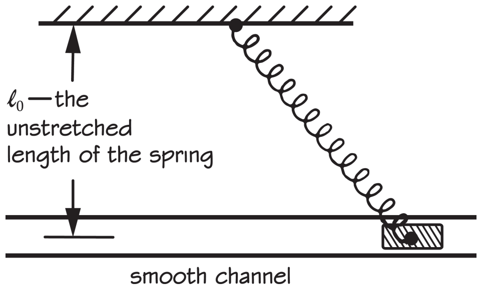
Isothermal Process

For the isothermal process, $\Delta Q = \Delta W$ because internal energy stays constant as long as temperature stays constant. Thus

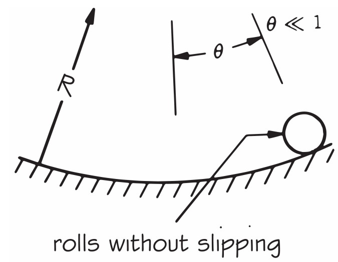
$$\Delta Q = nRT \ln\left(\frac{V_0}{2V_0}\right) = -270R\ln(2)$$

$$\begin{aligned} \Delta S &= \frac{\Delta Q}{270} \\ &= -R\ln(2) \\ &= -5.76\,\mathrm{J/K} \end{aligned}$$

(∗) 5. Determine which (if either) of the systems shown demonstrates simple harmonic motion. Why or why not?

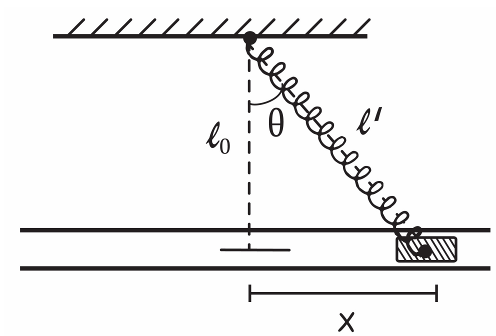


(a)

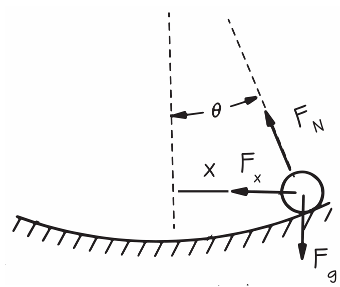


(b)

Solution: The following diagrams show the forces on the objects



(a)



(b)

We know that a mass will execute simple harmonic motion if the restoring force is linearly proportional to its displacement, i.e.

$$ma + kx = 0 \tag{5.1}$$

for mass m and some constant k. Therefore we will construct physical models of both scenarios, and check if the restoring force creates an equation in the form of (11.1).

(a) Let $|\mathbf{F}|$ be the magnitude of the spring force. Then, if we let

$$l' = \sqrt{l_0^2 + x^2} \tag{5.2}$$

where x is the horizontal displacement from equilibrium, we see that $|\mathbf{F}| = k(l' - l_0)$. ($k$ is the spring constant)
If we let θ be the angle between the vertical and the spring, then

$$\begin{aligned} F_x &= -|\mathbf{F}| \sin\theta & (5.3) \\ &= -k(l' - l_0)\sin\theta & (5.4) \end{aligned}$$

We will not consider forces in the vertical direction because the mass is constrained to move only in the horizontal direction.

Since $\sin\theta = x/l'$, we can plug it into (11.4) to obtain

$$F_x = -\frac{k(l' - l_0)x}{l'} \tag{5.5}$$

$$= -\frac{k(\sqrt{l_0^2 + x^2} - l_0)}{\sqrt{l_0^2 + x^2}}x \tag{5.6}$$

This expression is clearly not linear with respect to x. Thus the object *will not* execute simple harmonic motion.

(b) If we let x be the horizontal distance from equilibrium, then $\sin\theta = x/R$. Since the angle θ is small, the vertical acceleration is negligible (almost 0). This means that the normal force $|\mathbf{N}| \approx mg$.

$$F_x = -|\mathbf{N}|\sin\theta \tag{5.7}$$

$$= -\frac{mgx}{R} \tag{5.8}$$

Thus the mass will execute simple harmonic motion for small angles because the restoring force is linear in x (for small angles).

(**) 6. A gas of coefficient γ in a cylinder of volume V_0 at temperature T_0 and pressure P_0 is compressed slowly and adiabatically to volume $V_0/2$. After being allowed to come to temperature equilibrium (T_0) at this volume, the gas is then allowed to expand slowly and isothermally to its original volume V_0. In terms of P_0, V_0, γ, what is the net amount of work W the piston does on the gas?

Solution: The work is done in two processes. First the adiabatic compression, then the isothermal expansion.

Adiabatic Compression

The work for an adiabatic process is

$$W = \frac{C}{1-\gamma}(V_2^{1-\gamma} - V_1^{1-\gamma}),$$

where $C = PV^\gamma$, with values of P and V at any particular time during the process. For convenience we choose $C = P_0V_0^\gamma$.

Plugging in $V_2 = V_0/2$ and $V_1 = V_0$ we obtain

$$W = \frac{P_0V_0^\gamma}{1-\gamma}\left(\frac{V_0^{1-\gamma}}{2^{1-\gamma}} - V_0^{1-\gamma}\right) \quad (6.1)$$

$$= \frac{P_0V_0}{1-\gamma}(2^{\gamma-1} - 1) \quad (6.2)$$

Note that this work is the work done by the gas, and thus is negative. Since we are looking for the work done by the piston, we instead want the negative of (4.2), i.e.

$$W_1 = \frac{P_0V_0}{\gamma-1}(2^{\gamma-1} - 1) \quad (6.3)$$

Isothermal Expansion

For the expansion (isothermal), the work is given by

$$W = nRT\ln(V_f/V_i)$$

Plugging in the given values, we obtain

$$W = nRT_0\ln(V_0/(V_0/2))$$

$$= P_0V_0\ln(2)$$

Note that this work is positive because the gas did positive work. Since we are looking for the work done by the piston, we want

$$W_2 = -P_0V_0\ln(2) \quad (6.4)$$

Adding it up

Adding these two works together we obtain the work done by the piston on the gas:

$$\sum W = W_1 + W_2 \quad (6.5)$$

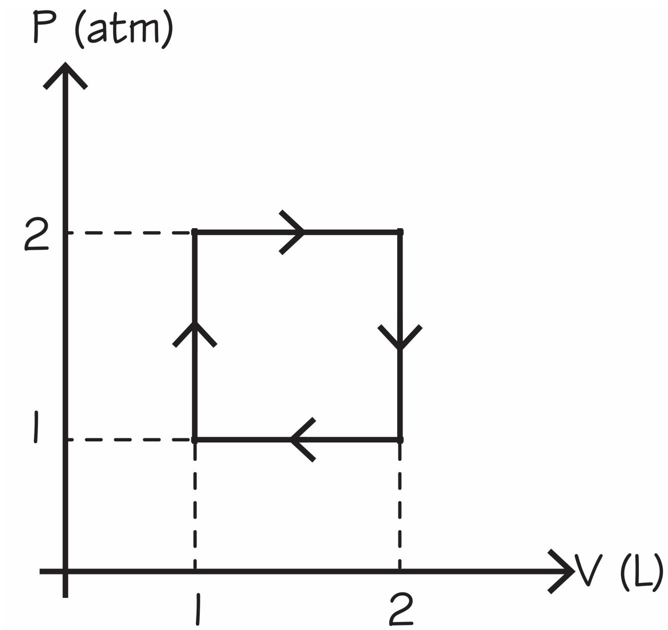
$$= P_0V_0\left(\frac{2^{\gamma-1}-1}{\gamma-1} - \ln(2)\right) \quad (6.6)$$

(∗∗) 7. An ideal gas with coefficient γ, is initially at the condition $P_0 = 1$ atm, $V_0 = 1$ litre, $T_0 = 300$ K. It is then:

(i) Heated at constant V until $P = 2$ atm.
(ii) Expanded at constant P until $V = 2$ litres.
(iii) Cooled at constant V until $P = 1$ atm.
(iv) Contracted at constant P until $V = 1$ litre.

(a) Draw a P–V diagram for this process.
(b) What work W is done per cycle?
(c) What is the maximum temperature T_{max} the gas attains?
(d) What is the total heat input ΔQ in steps (i) and (ii) in terms of γ?

Solution:



(a) See figure for P–V diagram.

(b) For an isochoric process (processes (i), (iii)) there is no work done. Therefore we just sum up the works from processes (ii) and (iv). For process (ii), work is given by

$$W_{\text{ii}} = P\Delta V = 2(2-1) = 2\,\text{atm}\cdot\,\text{litre} = 202.6\,\text{J}$$

For process (iv), work is given by

$$W_{\text{iv}} = P\Delta V = 1(1-2) = -1\,\text{atm}\cdot\,\text{litre} = -101.3\,\text{J}$$

Summing up the works, we obtain

$$\begin{aligned} W &= W_{\text{i}} + W_{\text{ii}} + W_{\text{iii}} + W_{\text{iv}} \\ &= 0 + 202.6 + 0 - 101.3 \\ &= 101.3\,\text{J/cycle} \end{aligned}$$

(c) Maximum temperature will be attained when the gas is at largest volume and highest pressure. This will be after process ii (when $P = 2$ and $V = 2$). Since

$$\frac{PV}{T} = \text{constant}$$

we can calculate the constant with the initial temperature, pressure, and volume values as

$$\frac{P_0V_0}{T_0} = \frac{1}{300}$$

Plugging in the values $P = 2$ and $V = 2$, we obtain

$$\frac{1}{300} = \frac{2 \cdot 2}{T_{max}}$$

Thus

$$T_{max} = 1200\,\text{K}$$

(d) We can use the specific heat capacity equations to find the total heat input. Process i is isochoric, so

$$Q_\text{i} = nC_V\Delta T$$

Process ii is isobaric, so

$$Q_\text{ii} = nC_P\Delta T'$$

The changes in temperature for each process are

$$\Delta T_\text{i} = 300\,\text{K}$$

and

$$\Delta T_\text{ii} = 600\,\text{K}$$

(From the ideal gas law). Thus

$$\begin{aligned} \Delta Q &= Q_\text{i} + Q_\text{ii} && (7.1)\\ &= nC_V\Delta T_\text{i} + nC_p\Delta T_\text{ii} && (7.2)\\ &= 300nC_V + 600nC_P && (7.3) \end{aligned}$$

We want to find a value for n to plug into (5.3). From the ideal gas law,

$$P_0V_0 = nRT_0 = n(C_P - C_V)T_0 \tag{7.4}$$

we solve for

$$n = \frac{P_0V_0}{T_0(C_P - C_V)} = \frac{101.3}{300(C_P - C_V)} \tag{7.5}$$

Plugging (5.5) into the expression for ΔQ (5.3), we obtain

$$\begin{aligned} \Delta Q &= 300C_V\frac{101.3}{300(C_P - C_V)} + 600C_P\frac{101.3}{300(C_P - C_V)} && (7.6)\\ &= 101.3\left(\frac{C_V}{C_P - C_V} + 2\frac{C_P}{C_P - C_V}\right) && (7.7)\\ &= 101.3\left(\frac{2C_P + C_V}{C_P - C_V}\right) && (7.8) \end{aligned}$$

Dividing numerator and denominator by C_V, we obtain

$$\Delta Q = 101.3\left(\frac{2\gamma + 1}{\gamma - 1}\right)\ \text{J}$$

(**) 8. The first Earth settlers on the moon will have great problems in keeping their living quarters at a comfortable temperature. Consider the use of Carnot engines for climate control. Assume that the temperature during the moon-day is 100°C, and during the moon-night is −100° C The temperature of the living quarters is to be kept at 20°C. The heat conduction rate through the walls of the living quarters is 0.5 kW per degree of temperature difference.

(a) Find the power P_{day} which has to be supplied to the Carnot engine during the day, and

(b) the power P_{night} which must be supplied at night.

Solution:

(a) During the daytime, the Carnot engine will be run in *reverse*, because it is cooling (refrigerating) the living quarters. This means that we put in work, and heat is taken from a cold reservoir (indoors) and sent to a warm reservoir (outdoors).

The equation for coefficient of performance for a refrigerator is

$$COP = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$$

In a Carnot engine,

$$\frac{|Q_C|}{|Q_H|} = \frac{|T_C|}{T_H|}$$

Thus we arrive at the coefficient of performance for a Carnot refrigerator,

$$\begin{aligned} COP &= \frac{|Q_C|}{|Q_H| - |Q_C|} && (8.1) \\ &= \frac{|Q_C|/|Q_H|}{1 - |Q_C|/|Q_H|} && (8.2) \\ &= \frac{T_C}{T_H - T_C} && (8.3) \end{aligned}$$

Another way to write the coefficient of performance is

$$COP = \frac{|Q_C|}{|W|} = \frac{|Q_C|/t}{|W|/t} = \frac{H}{P} \quad (8.4)$$

where H is the heat current through the walls of the living quarters and P is the power supplied to the engine.

For the daytime, we have $T_C = 293$ K, and $T_H = 373$ K, and $H = 40$ kW. Combining (6.3) and (6.4), we obtain

$$\frac{H}{P_{\text{day}}} = \frac{T_C}{T_H - T_C} \quad (8.5)$$

Plugging in the appropriate variables into (6.5) and solving for P_{day},

$$P_{\text{day}} = 10.9\,\text{kW}$$

(b) For the night time, we have $T_C = 173$ K, and $T_H = 293$ K, and $H = 60$ kW. Because we want to heat the interior this time, we will run the Carnot engines forward.

We have two expressions for efficiency:

$$e = 1 - \frac{T_C}{T_H} = 0.41 \tag{8.6}$$

$$e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t} = \frac{P}{H} \tag{8.7}$$

Equating (6.6) and (6.7),

$$\frac{P_{\text{night}}}{H} = 1 - \frac{T_C}{T_H} = 0.41 \tag{8.8}$$

Solving for P_{night},

$$P_{\text{night}} = 24.6\,\text{kW}$$

(∗∗) 9. Two samples of gas, A and B of the same initial volume V_0, and at the same initial absolute pressure P_0, are suddenly compressed adiabatically, each to one half its initial volume.

(a) Express the final pressures (P_A, P_B) of each sample in terms of the initial pressure P_0, if $\gamma_A = 5/3$ (monatomic) and $\gamma_B = 7/5$ (diatomic)

(b) Find the ratio of work W_A/W_B required to perform the two compressions described.

Solution:

(a) We have, for an adiabatic process,

$$P_0 V_0^{\gamma} = P_A V_A^{\gamma_A} = P_B V_B^{\gamma_B} \tag{9.1}$$

Since both containers are compressed to one-half of the original volume,

$$V_A = V_B = V_0/2$$

We obtain

$$P_0 V_0^{\gamma_A} = P_A \left(\frac{V_0}{2}\right)^{\gamma_A}$$

$$P_0 V_0^{\gamma_B} = P_B \left(\frac{V_0}{2}\right)^{\gamma_B}$$

Thus

$$(P_A, P_B) = (2^{\gamma_A} P_0, 2^{\gamma_B} P_0) = (3.17P_0, 2.64P_0)$$

(b) For an adiabatic process, the formulas for work are:

$$W_A = \frac{P_0 V_0^{\gamma_A}}{1-\gamma_A}(V_A^{1-\gamma_A} - V_0^{1-\gamma_A}) \tag{9.2}$$

$$W_B = \frac{P_0 V_0^{\gamma_B}}{1-\gamma_B}(V_B^{1-\gamma_B} - V_0^{1-\gamma_B}) \tag{9.3}$$

Since

$$V_A = V_B = V_0/2$$

we can plug $V_A = V_0/2$ into (9.2):

$$W_A = \frac{P_0 V_0^{\gamma_A}}{1-\gamma_A}\left[\left(\frac{V_0}{2}\right)^{1-\gamma_A} - V_0^{1-\gamma_A}\right] \tag{9.4}$$

$$= \frac{P_0 V_0}{1-\gamma_A}\left(\frac{1}{2^{1-\gamma_A}} - 1\right) \tag{9.5}$$

and $V_B = V_0/2$ into (9.3):

$$W_B = \frac{P_0 V_0^{\gamma_B}}{1-\gamma_B}\left[\left(\frac{V_0}{2}\right)^{1-\gamma_B} - V_0^{1-\gamma_B}\right] \tag{9.6}$$

$$= \frac{P_0 V_0}{1-\gamma_B}\left(\frac{1}{2^{1-\gamma_B}} - 1\right) \tag{9.7}$$

Dividing (9.5) by (9.7), we obtain

$$\begin{aligned}\frac{W_A}{W_B} &= \frac{1-\gamma_B}{1-\gamma_A} \cdot \left(\frac{\frac{1}{2^{(1-\gamma_A)}} - 1}{\frac{1}{2^{(1-\gamma_B)}} - 1} \right) \\ &= \frac{1-\gamma_B}{1-\gamma_A} \cdot \left(\frac{2^{\gamma_A - 1} - 1}{2^{\gamma_B - 1} - 1} \right) \\ &= 1.1\end{aligned}$$

(∗∗) 10. Two particles A and B execute harmonic motion of the same amplitude (10 cm) on the same straight line. For particle A, $\omega_A = 20$ rad/s; for B, $\omega_B = 21$ rad/s. If at $t = 0$, they both pass through $x = 0$ in the positive x-direction (hence both of them are "in phase")

(a) How far apart, Δx will they be at $t = 0.350$ s?

(b) What is the velocity V of B relative to A at $t = 0.350$ s?

(c) How long after $t = 0$ does it take for them to *both* be at $x = 0$ at the same time again?

Solution:

(a) Since we know that the equation of motion of a harmonic oscillator is

$$x(t) = A\cos(\omega t + \phi) \tag{10.1}$$

we can use this to find equations of motion for particles A and B.

- At $t = 0$, both particles are at 0. They also both have positive velocities at this instant. This means that we should choose a sine instead of a cosine, and set $\phi = 0$.
- The amplitudes of both are 10, so choose $A = 10$ for both motions.
- The angular frequencies are given as $\omega_A = 20$ rad/s, and $\omega_B = 21$ rad/s.

Thus

$$x_A(t) = 10\sin(20t) \tag{10.2}$$
$$x_B(t) = 10\sin(21t) \tag{10.3}$$

To find the distance between, Δx between them at $t = 0.35$, we take

$$\begin{aligned}\Delta x &= |x_B(0.35) - x_A(0.35)| \\ &= |8.76 - 6.57| \\ &= 2.19\,\text{cm}\end{aligned}$$

(b) We must differentiate to find the equations for velocities.

$$x'_A(t) = v_A(t) = 200\cos(20t) \tag{10.4}$$
$$x'_B(t) = v_B(t) = 210\cos(21t) \tag{10.5}$$

The velocity of B relative to A at $t = 0.35$ is

$$\begin{aligned}V &= v_B(0.35) - v_A(0.35) \\ &= 101.4 - 150.8 \\ &= -49.4\,\text{cm/s}\end{aligned}$$

(c) We set

$$\begin{aligned}x_A &= x_B = 0 \\ 10\sin(20t) &= 10\sin(21t) \\ \sin(20t) &= \sin(21t) = 0\end{aligned}$$

Thus $\sin(20t) = 0$ if $t = n\pi/20$, and $\sin(21t') = 0$ if $t' = m\pi/21$ for some integers n, m.

Setting $t = t'$, we obtain

$$n\pi/20 = m\pi/21$$

This means that

$$21n = 20m$$

The smallest solution to this equation is $n = 20$, $m = 21$ because 20 and 21 share no common factors.

Thus

$$t = 20\pi/20 = \pi\,\mathrm{s}$$

will be the first time after $t = 0$ that both objects are at $x = 0$ simultaneously.

(**) 11. A 20 g hook with a 5 g weight on it is attached to a vertical spring of negligible mass. When the spring is displaced from equilibrium the system is found to oscillate in vertical simple harmonic motion with a period of $\pi/3$ s. If the 5 g weight is replaced by a 25 g weight, how far z can the spring be displaced from equilibrium before release, if the weight is not to jump off the hook?

Solution: If the period $T = \pi/3$, then that means

$$T = \frac{\pi}{3} = 2\pi\sqrt{\frac{m}{k}} \tag{11.1}$$

The mass in the first case was, in total, 25 g, so that gives $k = 0.9$ N/m. Now we can calculate the angular frequency ω for the case when the total mass is 45 g.

$$\omega = \sqrt{\frac{k}{m}} \tag{11.2}$$

$$= \sqrt{\frac{0.9}{0.045}} \tag{11.3}$$

$$= \sqrt{20} \tag{11.4}$$

$$= 4.47\,\text{rad/s} \tag{11.5}$$

The mass will jump off if the downward acceleration of the oscillator exceeds $g = 9.8\,\text{m/s}^2$.

The maximum acceleration will be $z\omega^2$, where z is the amplitude of the *motion*. Thus

$$z\omega^2 = g \tag{11.6}$$

$$20z = 9.8 \tag{11.7}$$

$$z = 0.49\,\text{m} \tag{11.8}$$

Thus $z = 49$ cm.

(∗ ∗ ∗) 12. In an ideal reversible engine employing 28 g nitrogen as working substance ($\gamma = 7/5$) in a cyclic operation $a \to b \to c \to d$ without valves, the temperature of the source is 400 K, and the temperature of of the sink is 300 K. The initial volume of gas at point a is 6.0 litres and the volume at point c is 18.0 litres.

(a) At what volume V_b should the cylinder be changed from heat input (isothermal expansion) to isolation and adiabatic expansion (from V_b to V_c)?

(b) At what volume V_d should the adiabatic compression begin?

(c) How much heat $\Delta Q_{a\to b}$ is put in during the $V_a \to V_b$ part of the cycle?

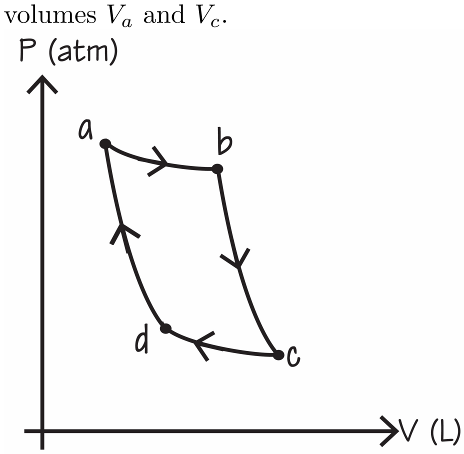
(d) How much heat $\Delta Q_{c\to d}$ is extracted during the $V_c \to V_d$ part?

(e) What is the efficiency e of the engine?

(f) What change ΔS in entropy per gram occurs in the working substance during $a \to b$ and $c \to d$?

Hint. *For a Carnot cycle the expansion ratios V_b/V_a and V_c/V_d are equal. Draw yourself a P–V diagram to help understand the cycle.*

Solution: Since the question says an ideal reversible engine, this implies that we should set up a Carnot cycle. First, we draw a P–V diagram to help us understand the cycle. We know that the temperature at the upper isotherm is 400 K, and that the temperature at the lower isotherm is 300 K. We also know that a and c have to go in the places they do because the given values tell us the volumes V_a and V_c.



We will first write down all of our known relations, even though we will not need to use some of them. Since we are working with 28 g of Nitrogen, and nitrogen is element 7, 28 grams of nitrogen means 1 mol of nitrogen, so $n = 1$. We also know the volumes of $V_a = 6\,\text{L}$, and $V_c = 18\,\text{L}$.
For the ideal gas law at each point, we have (because $n = 1$)

$$P_aV_a = RT_a \tag{12.1}$$
$$P_bV_b = RT_b \tag{12.2}$$
$$P_cV_c = RT_c \tag{12.3}$$
$$P_dV_d = RT_d \tag{12.4}$$

For the two adiabatic processes, we have

$$T_cV_c^{\gamma-1} = T_bV_b^{\gamma-1} \tag{12.5}$$
$$T_aV_a^{\gamma-1} = T_dV_d^{\gamma-1} \tag{12.6}$$

For the two isothermal processes, we have

$$\begin{aligned} P_aV_a &= P_bV_b \quad (12.7) \\ P_cV_c &= P_dV_d \quad (12.8) \end{aligned}$$

Because this is a Carnot cycle, we have

$$V_aV_c = V_dV_b \quad (12.9)$$

Some known values are:

$$T_a = T_b = 400\,\text{K}$$

$$T_c = T_d = 300\,\text{K}$$

(a) Plugging in $T_c = 300$, $T_b = 400$, $V_c = 18$ into (8.5) yields

$$300V_c^{\gamma-1} = 400V_b^{\gamma-1}$$

$$300(18^{7/5-1}) = 953.3$$

$$V_b^{2/5} = 2.38$$

$$V_b = 8.8\,\text{L}$$

(b) Plugging in $T_d = 300$, $T_a = 400$, $V_a = 6$ to (8.6) yields

$$400V_a^{\gamma-1} = 300V_d^{\gamma-1}$$

$$400(6^{7/5-1}) = 819$$

$$V_d^{2/5} = 2.73$$

$$V_d = 12.3\,\text{L}$$

(c) The process $a \to b$ is isothermal, so the change in internal energy is 0. By the first law of thermodynamics, we then have $\Delta U = 0 = \Delta Q - W$. Thus

$$\begin{aligned} \Delta Q_{a\to b} &= W \\ &= nRT_a \ln(V_b/V_a) \\ &= 400R\ln(8.8/6) \\ &= 1.26 \cdot 10^3\,\text{J} \end{aligned}$$

(d) The process $c \to d$ is isothermal, so the change in internal energy is 0. By the first law of thermodynamics, we then have $\Delta U = 0 = \Delta Q - W$. Thus

$$\begin{aligned} \Delta Q_{c\to d} &= W \\ &= nRT_c \ln(V_d/V_c) \\ &= 300R\ln(12.3/18) \\ &= -945.9\,\text{J} \end{aligned}$$

This means that +945.9 J were removed.

(e) The efficiency of the engine will be the Carnot efficiency, which is

$$e = 1 - \frac{T_C}{T_H} = 1 - 0.75 = 0.25$$

Thus the engine is 25% efficient.

(f) During $a \to b$ (or $c \to d$, doesn't matter, it is the same ΔS), we have

$$\begin{aligned} \Delta S &= \frac{\Delta Q_{a\to b}}{T_a} \\ &= \frac{1.26 \cdot 10^3}{400} \\ &= 3.15\,\text{J/K} \end{aligned}$$

To find the entropy per gram, divide by the molar mass of N_2. Per gram, that is $3.15/28 = 0.11$ J/gK

Remark. *If we had instead used* $\Delta Q_{c\to d}/T_c = 945/300 = 3.15$ *we would have gotten the same answer*

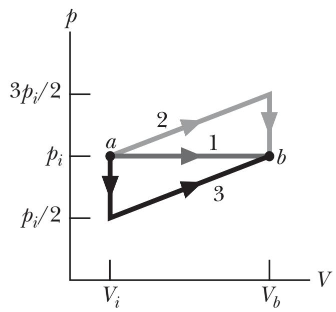
(∗ ∗ ∗) 13. A sample of gas undergoes a transition from an initial state a to a final state b by three different paths, as shown in the P-V diagram, where $V_b = 5.00V_i$. The energy transferred to the gas as heat in process 1 is $10P_iV_i$.

(a) How many degrees of freedom does the sample of gas have?

(b) Find the energy transferred to the gas as heat in process 2.

(c) Find the change in internal energy that the gas undergoes in process 3.

Express your answers in terms of P_i, V_i.



Solution:

(a) We will first try and get more data about path 1. Since it is an isobaric process because pressure is constant, the work done will be

$$W = P_i(V_b - V_i) = 4P_iV_i$$

From the ideal gas law $V_i/T_i = V_b/T_b$, we can find that

$$T_b = 5T_i$$

$$\Delta T = 4T_i$$

The change in internal energy will be

$$\begin{aligned} \Delta U &= nC_v\Delta T \\ &= nC_V(4T_i) \\ &= 4C_V(nT_i) \\ &= 4C_V(P_iV_i/R) \end{aligned}$$

Then we can use the first law of thermodynamics to find a relationship between C_V and R.

$$\Delta U = Q - W$$

$$4C_VP_iV_i/R = 10P_iV_i - 4P_iV_i = 6P_iV_i$$

$$C_V = 3R/2$$

Thus we conclude that it is a monatomic gas and has 3 degrees of freedom. Note that the three degrees of freedom are simply movement in the x, y, and z directions.

(b) The work for the first segment of path 2 can be found using the area under the line. We use the formula for the area of a trapezoid.

$$W = (V_b - V_i)(P_i + 3P_i/2)/2 = 5P_iV_i \tag{13.1}$$

The change in temperature in the first segment of path 2 can be found using the ideal gas law. We have

$$P_iV_i/T_i = P_bV_b/T_b(5V_i)(3P_i/2)/(T)$$
$$T = 15T_i/2$$

This means that $\Delta T = 13/2T_i$.

The change in internal energy will be

$$\begin{aligned}\Delta U &= nC_V(13/2T_i) &(13.2)\\ &= (13/2)C_VP_iV_i/R &(13.3)\\ &= (13/2)(3/2)P_iV_i &(13.4)\\ &= 39P_iV_i/4 &(13.5)\end{aligned}$$

We can use the first law of thermodynamics to find that $\Delta Q = \Delta U + W$ for this first segment of the path (using (14.1) and (14.5)) is

$$39P_iV_i/4 + 5P_iV_i = 59P_iV_i/4 \tag{13.6}$$

For the second (vertical) segment of the path, there is no work done. Thus the heat added (at constant volume) will depend only on the temperature difference. We know that the temperature at b is $5T_i$, so then the change in heat energy for this second segment will be

$$Q = nC_V(-5/2T_i) = -15P_iV_i/4 \tag{13.7}$$

Adding these two heats, we find that the total change in heat energy over path 2 is

$$\Delta Q = 59P_iV_i/4 - 15P_iV_i/4 = 11P_iV_i$$

(c) Internal energy is independent of path, so then

$$\Delta U = nC_V(4Ti) = 6P_iV_i$$

(∗ ∗ ∗) 14. An insulated container with a movable, frictionless piston of mass M and area A, contains N grams of helium gas in a volume V_1, as shown. The external pressure is P. The gas is *very slowly* heated by an internal heating coil until the volume occupied by the gas is $2V_1$. What is,

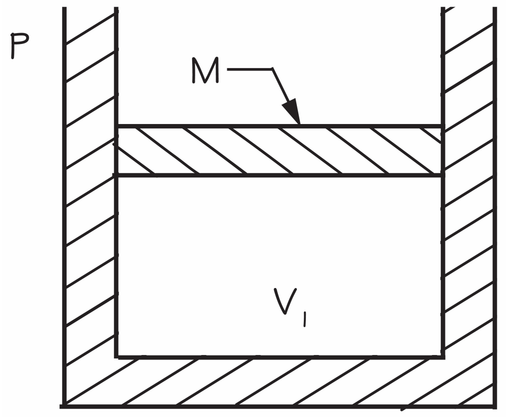
(a) the work W done by the gas?

(b) the heat ΔQ supplied to the gas?

(c) the change ΔU in the internal energy of the gas?

(d) the initial temperature T_i and the final temperature T_f of the gas?

Express your answers in terms of the given variables M, A, P, N, V_1.



Solution:

(a) The piston has area A, so then we know that

$$V_1 = hA$$

where h is the distance between the bottom of the container and the piston. Since the piston is moved so that the container is $2V_1$ in volume, we conclude that the piston moved up a distance of h to a new height of $2h$.
The forces on the piston are due to gravity, external pressure P, and the pressure of the gas inside. The gas must work against the force of gravity and the external pressure. Those two forces are given by $PA + Mg$. Thus the work is

$$W = (PA + Mg)h \quad (14.1)$$
$$= \frac{(PA + Mg)V_1}{A} \quad (14.2)$$
$$= \left(P + \frac{Mg}{A}\right) V_1 \quad (14.3)$$

(b) The piston moves very slowly. This means that the pressure inside is roughly constant throughout the expansion. Thus

$$\Delta Q = nC_P \Delta T \quad (14.4)$$
$$= nC_P \left(\frac{P\Delta V}{nR}\right) \quad (14.5)$$
$$= \frac{5P\Delta V}{2} \quad (14.6)$$
$$= \frac{5}{2}W \quad (14.7)$$
$$= \frac{5}{2}\left(P + \frac{Mg}{A}\right) V_1 \quad (14.8)$$

(c) From the first law of thermodynamics,

$$\begin{aligned}\Delta U &= \Delta Q - W && (14.9)\\ &= \frac{5W}{2} - W && (14.10)\\ &= \frac{3W}{2} && (14.11)\\ &= \frac{3}{2}\left(P + \frac{Mg}{A}\right) V_1 && (14.12)\end{aligned}$$

(d) Since the forces on the piston must have been balanced at the start, we have, for some initial pressure P_i,

$$P_i A = PA + Mg \quad (14.13)$$

$$P_i = P + \frac{Mg}{A} \quad (14.14)$$

From the ideal gas law, we have

$$T_i = \frac{P_i V_i}{nR} \quad (14.15)$$

Since the gas is helium, we know that there are 4 grams per mol, which means that the number of moles n is given by $n = N/4$. Thus, plugging (14.14) into the ideal gas law (14.15),

$$\begin{aligned}T_i &= \frac{4V_1}{NR}\left(P + \frac{Mg}{A}\right) && (14.16)\\ &= \frac{4W}{NR} && (14.17)\end{aligned}$$

We know that the change in internal energy is

$$\Delta U = \frac{3W}{2} \quad (14.18)$$

and since

$$\Delta U = nC_V \Delta T \quad (14.19)$$

we can equate (14.18) and (14.19) to obtain

$$\frac{3W}{2} = \frac{N}{4} \cdot \frac{3R}{2}(T_f - T_i) \quad (14.20)$$

Multiplying both sides of (14.20) by $8/(3NR)$,

$$\frac{4W}{NR} = T_f - T_i = T_i$$

Thus

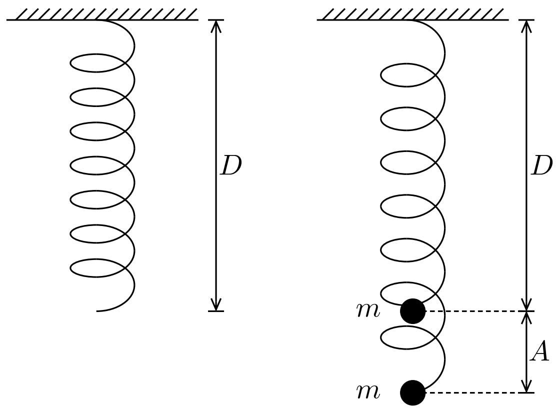
$$\begin{aligned}T_f &= 2T_i\\ &= \frac{8V_1}{NR}\left(P + \frac{Mg}{A}\right)\end{aligned}$$

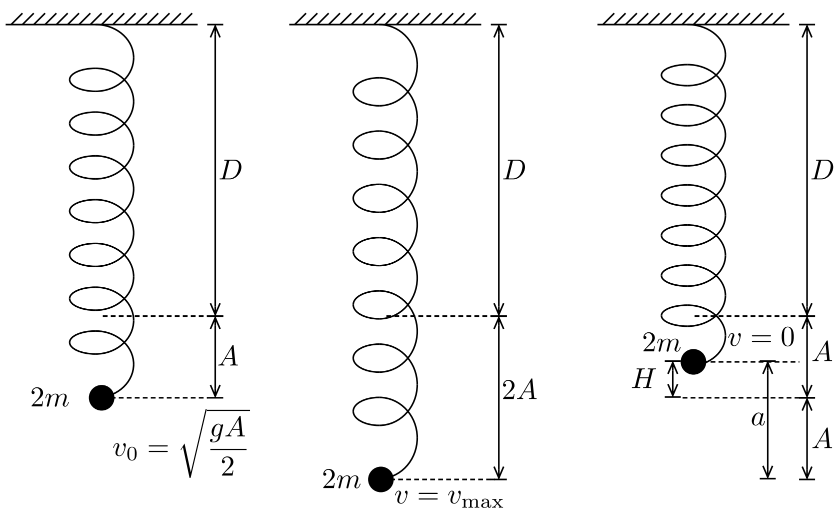
(∗ ∗ ∗) 15. A certain linear spring has a free length D. When a mass m is hung on the end, it has a length $D + A$. While it is hanging motionless with mass m attached, a second mass m is dropped from a height A onto the first one, with which it collides inelastically (i.e. they stick together). For the resulting motion, find the:

(a) period T

(b) amplitude a, and

(c) maximum height H (above the original equilibrium position)





Solution:

(a) We know that $\omega = \sqrt{k/m}$, and $T = 2\pi/\omega$. Performing a force balance on the mass m after it is placed on the spring, we can calculate the spring constant k in terms of the given variables. Since the spring extended by a length A when a mass m was put on it, we know that

$$mg = kA \tag{15.1}$$

then rearranging:

$$k = \frac{mg}{A}. \tag{15.2}$$

The resulting motion in fact has $2m$ oscillating, but the spring constant is still the same. Combining the above equation with the definition of angular frequency,

$$\omega = \sqrt{\frac{g}{2A}} \tag{15.3}$$

and rearranging yields

$$T = 2\pi\sqrt{\frac{2A}{g}} \tag{15.4}$$

(b) The first thing to note is that there will be a *new* equilibrium position now that a second mass has been added on top of the previous one. The new mass is $2m$, so the extension of the spring h at the equilibrium height can be calculated by

$$2mg = kh = (mg/A)h$$

which gives us $h = 2A$. Thus the equilibrium length of the spring is $D + 2A$.

Now we need to find the velocity of the system when the two masses collide. To do this we can apply the conservation of momentum. When the mass is dropped from a height A it will lose potential energy mgA, and thus will have velocity

$$v = \sqrt{2gA} \tag{15.5}$$

when it strikes the other mass. By the conservation of momentum, we have

$$mv = (m + m)v_0 \tag{15.6}$$

where v_0 is the initial velocity of the combined mass ($2m$). We can then calculate the initial velocity of the combined mass:

$$v_0 = \sqrt{gA/2} \tag{15.7}$$

To solve the remainder of the question, we will apply the conservation of energy. Since the masses collide at height A above the new equilibrium point, their total energy will be

$$E_0 = \frac{2mv_0^2}{2} + \frac{kA^2}{2} \tag{15.8}$$

(kinetic energy + spring potential energy). Note that there is *no* gravitational potential energy term here because that has already been accounted for by the new equilibrium point (measuring spring displacements from $D + 2A$ instead of displacements from D). When the masses have zero velocity, they will have the maximum displacement from equilibrium. Thus (again ignoring gravitational potential) conservation of energy will give us

$$\frac{2mv_0^2}{2} + \frac{kA^2}{2} = \frac{ka^2}{2} \tag{15.9}$$

Thus plugging in v_0 and k into the above equation and solving for a,

$$m\left(\frac{gA}{2}\right) + \left(\frac{mg}{A}\right)\frac{A^2}{2} = \left(\frac{mg}{A}\right)\frac{a^2}{2} \tag{15.10}$$

$$2A = \frac{a^2}{A} \tag{15.11}$$

$$a = A\sqrt{2} \tag{15.12}$$

(c) With current equilibrium point (with two masses) at $D + 2A$, initial equilibrium point (with only one mass) at $D + A$, and amplitude $a = A\sqrt{2}$, we have the maximum height at

$$D + 2A - \sqrt{2}A = D + A(2 - \sqrt{2})$$

To find how high this is above the initial equilibrium point of $D + A$, we take the difference:

$$D + A - [D + A(2 - \sqrt{2})] = A(\sqrt{2} - 1)$$

Thus

$$H = A(\sqrt{2} - 1)$$

Remark. If you're uncomfortable with ignoring the gravitational potential energy in part (b), we will provide a justification here. Suppose we take the gravitational potential energy with respect to the point where the spring is stretched to $D + 2A$. We must also then use the *original* spring length as the one we're taking reference to. That is, we're measuring how far the spring is stretched from D, as opposed to how far it is stretched from $D + 2A$, which is what we measured in part (b). Then the total energy starts off as

$$E_0 = \frac{(2m)v_0^2}{2} + \frac{kA^2}{2} + (2m)gA$$

At the point of zero velocity, we have that the energy is

$$E = \frac{k(2A - a)^2}{2} + (2m)ga$$

Then setting these two expressions equal,

$$\begin{aligned}
mv_0^2 + \frac{kA^2}{2} + (2m)gA &= \frac{k}{2}(2A - a)^2 + (2m)ag \\
\frac{mgA}{2} + \frac{mgA}{2} + 2mgA &= \frac{mg}{2A}(2A - a)^2 + 2mag \\
3mgA &= \frac{mg}{2A}(2A - a)^2 + 2mag \\
3A &= \frac{1}{2A}(4A^2 - 4aA + a^2) + 2a \\
6A^2 &= 4A^2 - 4aA + a^2 + 4aA \\
2A^2 &= a^2 \\
\sqrt{2}A &= a
\end{aligned}$$

and we see that we arrive at the same answer for the new amplitude.

Useful Constants and Conversion Ratios:

R = Ideal Gas constant = 8.31451 J/molK, 1 atm = 1.013×10^5 Pa, 1 atm · litre = 101.3 J

σ = Stefan-Boltzmann constant = 5.6704×10^{-8} W/m^{2}K^4, $\gamma_{\text{air}} = 1.4$, $C_{V_{\text{air}}} = 20.8$ J/molK

ρ_{water} = Density of water = 1 gram/cm^3 = 1000 kg/m^3

Mechanics:

Linear Motion: $x = x_0 + \frac{1}{2}(v_0 + v)t$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2a(x - x_0)$

Circular Motion: $a_c = \dfrac{v^2}{r}$

Forces: $\mathbf{F} = m\mathbf{a} = \dfrac{d}{dt}\mathbf{p}$, Friction: $|\mathbf{F}| = \mu|\mathbf{N}|$, Spring: $\mathbf{F} = -k\mathbf{x}$, Damping: $\mathbf{F} = -b\mathbf{v}$

Buoyant $|\mathbf{F}| = \rho V g$

$W = \text{Work} = \displaystyle\int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r} = \mathbf{F} \cdot \Delta\mathbf{r}$, $K = \frac{1}{2}mv^2$, $\Delta U_{\text{gravity}} = mg\Delta h$, $\Delta U_{\text{spring}} = \frac{1}{2}kx^2$

$P = \dfrac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$

Thermodynamics:

Thermal Expansion: $\Delta L = \alpha L_0 \Delta T$, Stress and Strain: $\dfrac{|\mathbf{F}|}{A} = Y\dfrac{\Delta L}{L}$, Ideal Gas Law: $PV = nRT$

$K_{\text{av}} = \frac{3}{2}kT$

Thermal Conductivity: $I = \dfrac{\Delta Q}{\Delta t} = kA\dfrac{\Delta T}{\Delta x}$

Black Body Radiation: $P = e\sigma AT^4$, $\lambda_{\max}T = 2.8977685 \times 10^{-3} m \cdot K$

Internal Energy: $U = nC_V T$

First Law of Thermodynamics: $dQ = dU + dW$ For an ideal gas, $dW = PdV$

Work for an *isothermal process* $W = nRT\ln(V_f/V_i)$

Work for an *adiabatic expansion* $TV^{\gamma-1}$ = constant, if the number of moles is constant $PV^\gamma = C$ where C is a constant and $\gamma = C_P/C_V$

Work for adiabatic process: $W = \displaystyle\int_{V_1}^{V_2} PdV = C\int_{V_1}^{V_2} \frac{dV}{V^\gamma} = \frac{C}{1-\gamma}(V_2^{1-\gamma} - V_1^{1-\gamma})$

Heat Transfer: $Q = mc\Delta T$, $Q = mL$, $C_P = C_V + R$, $C_V = \dfrac{f}{2}R$, where f = degrees of freedom.

$f = 3$ for monatomic and $f = 5$ for diatomic.

$dS = \dfrac{dQ}{T}$

$e = W/Q_H$, $COP_{\text{Cooling}} = \dfrac{|Q_C|}{|W|}$, $COP_{\text{Heating}} = \dfrac{|Q_H|}{|W|}$, $e_{\text{Carnot}} = 1 - \dfrac{T_C}{T_H}$

Integrals:

$\displaystyle\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq 1$ $\displaystyle\int x^{-1}dx = \ln x + C$

Trigonometry:

$\sin\theta_1 + \sin\theta_2 = 2\cos\left(\dfrac{\theta_1 - \theta_2}{2}\right)\sin\left(\dfrac{\theta_1 + \theta_2}{2}\right)$

Area and Volume:

Surface Area of a sphere: $A = 4\pi r^2$. Lateral surface area of a cylinder: $A = 2\pi rl$.

Area of a circle: $A = \pi r^2$. Volume of a cylinder: $V = l\pi r^2$ Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Oscillations:

$\omega = 2\pi f$, $T = \dfrac{1}{f}$, $x = A\cos(\omega t + \phi)$, $\omega^2 = \dfrac{k}{m}$

Damped Oscillations: $x = A_0 e^{-\frac{bt}{2m}}\cos(\omega t + \phi)$, where $\omega = \sqrt{w_0^2 - \left(\dfrac{b}{2m}\right)^2}$, $Q = 2\pi\dfrac{E}{\Delta E}$

Energy for damped $E = E_0 e^{-\frac{bt}{m}}$

Waves:

$v = \sqrt{\frac{T}{\mu}}$, $k = \frac{2\pi}{\lambda}$, $P = \frac{1}{2}\mu\omega^2 A^2 v$, $p_o = \rho\omega v s_0$

$v = \sqrt{\frac{\gamma RT}{M}}$, $I = \frac{P_{\text{av}}}{4\pi r^2}$, $\beta = 10dB\log_{10}\left(\frac{I}{I_0}\right)$, Doppler Effect $f' = f_0\left(\frac{v \pm v_L}{v \mp v_S}\right)$

Beats: $\Delta f = f_2 - f_1$, $y = A\cos(kx \mp \omega t + \phi)$

Interference: $k\Delta x + \Delta\phi = 2\pi n$ or $\pi(2n+1)$, $n = 0, \pm1, \pm2, \pm3, \pm4, \ldots$

Standing Waves $f_m = \frac{mv}{2L}$, $m = 1, 2, 3, \ldots$, $f_m = \frac{mv}{4L}$, $m = 1, 3, 5, \ldots$

Constants:

$k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9\,\text{Nm}^2/\text{C}^2$, $\epsilon_0 = 8.84 \times 10^{-12}\,\text{C}^2/\text{Nm}^2$, $e = 1.6 \times 10^{-19}\,\text{C}$

$\mu_0 = 4\pi \times 10^{-7}\,\text{Tm/A}$, $c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 299,792,458\,\text{m/s}$

Point Charge:

$|\mathbf{F}| = \frac{k|q_1q_2|}{r^2}$, $|\mathbf{E}| = \frac{k|q|}{r^2}$, $V = \frac{kq}{r} + \text{Constant}$

Electric potential and potential energy $\Delta V = V_a - V_b = \int_a^b \mathbf{E}\cdot d\mathbf{l} = -\int_b^a \mathbf{E}\cdot d\mathbf{l}$

$E_x = -\frac{dV}{dx}$, $\mathbf{E} = -\nabla V$, $\Delta U = U_a - U_b = q(V_a - V_b)$

Maxwell's Equations:

$$\int_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} = 4\pi k Q_{\text{enc}} \qquad \int_S \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\int_C \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I_{\text{enclosed}}\right) + \epsilon_0\mu_0\frac{d\Phi_E}{dt} \qquad \int_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

Where S is a closed surface and C is a closed curve. $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A}$ and $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$

Energy Density:

$u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = \frac{1}{2\mu_0}B^2$ (energy per volume)

Forces:

$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$, $\mathbf{F} = I\mathbf{L}\times\mathbf{B}$

Capacitors:

$q = CV$, $U_C = \frac{1}{2}\cdot\frac{q^2}{C}$, For parallel plate capacitor with vacuum (air): $C = \frac{\epsilon_0 A}{d}$, $C_{\text{dielectric}} = KC_{\text{vacuum}}$

Inductors:

$\mathcal{E}_L = -L\frac{dI}{dt}$, $U_L = \frac{1}{2}LI^2$, where $L = N\Phi_B/I$ and N is the number of turns.

For a solenoid $B = \mu_0 nI$ where n is the number of turns per unit length.

DC Circuits: $V_R = IR$, $P = VI$, $P = I^2R$

(For RC circuits)$q = ae^{-t/\tau} + b, \tau = RC,$ a and b are constants

(For LR circuits)$I = ae^{-t/\tau} + b, \tau = L/R,$ a and b are constants

AC circuits: $X_L = \omega L$, $X_C = 1/(\omega C)$, $V_C = X_C I$, $V_L = X_L I$

$V = ZI$, $Z = \sqrt{(X_L - X_C)^2 + R^2}$, $P_{\text{average}} = I_{\text{rms}}^2 R$, $I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}}$

If $V = V_0\cos(\omega t)$, then $I = I_{\max}\cos(\omega t - \phi)$, where $\tan\phi = \frac{X_L - X_C}{R}$, $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$

Additional Equations: $d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3}$

LRC Oscillations: $q = A_0 e^{-\frac{Rt}{2L}}\cos(\omega t + \phi)$, where $\omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$ and $\omega_0^2 = \frac{1}{LC}$