

Physics 158 Midterm 1 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult.

Solutions posted at: <http://ubcengineers.ca/tutoring/>

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources (All solutions prepared by the EUS.):

- Electricity, Magnetism, and Light / Wayne Saslow
- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.

Want a warm up? These are the easier problems	Short on study time? These cover most of the material	Want a challenge? These are some tougher questions
1,2,3,4	3, 5, 6, 7	9, 10, 11

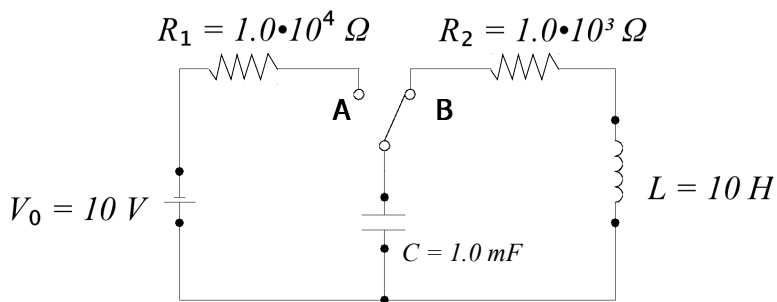
EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb won't help you understand the material.
- **Sleep**—We have all been told we need 8 hours of sleep a night, university shouldn't change this. Get to know how much sleep you need and set up a regular sleep schedule.



Good Luck!

- (*) 1. In the circuit shown in the figure, with the switch S open (so S begins neither at A nor B) at $t < 0$, the capacitor is uncharged.
- (a) After the switch is closed at A , how much time t does it take the voltage across the capacitor to reach 8.0 V?
- (b) At the instant the voltage across the C reaches 8.0 V, the switch is thrown to B . What is the initial value of the current I_0 that starts through L ?



Solution:

- (a) This is an RC circuit, so the equation governing the voltage across the capacitor is

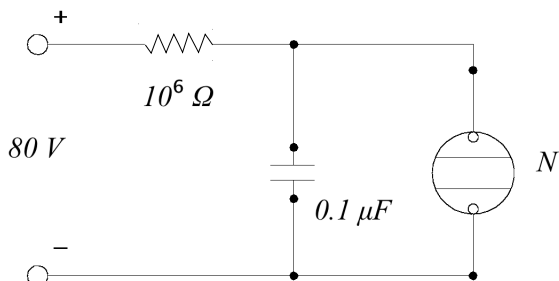
$$V(t) = V_0(1 - e^{-t/RC}) = 10(1 - e^{-t/10})$$

Plugging in $V = 8$, we obtain $t = 16.1 \text{ s}$.

- (b) The current through the inductor cannot change instantaneously, so

$$I_0 = 0$$

- (*) 2. The circuit shown in the figure constitutes what is called a *relaxation oscillator*. It consists of a neon bulb N connected parallel to a capacitor that is charged through a resistor from an 80 V DC voltage source. The neon tube has infinite resistance as long as the voltage across it is less than 60 V. If the voltage attained exceeds this value, the neon tube breaks down and then has negligible resistance, discharging the capacitor. The neon tube then "goes out", and returns to the infinite resistance state. If $C = 0.1 \mu\text{F}$, $R = 10^6 \Omega$, and $V_0 = 80 \text{ V}$, find the frequency f at which the neon tube flashes.



Solution: The time at which the first flash occurs will tell us the period of the flashing, which can then give us the frequency. The voltage across the capacitor is given by

$$V(t) = V_0(1 - e^{-t/RC}) = 80(1 - e^{-10t})$$

Setting $V(T) = 60$, we find

$$T = \text{period} = 0.139 \text{ s}$$

Thus,

$$f = 7.2 \text{ Hz}$$

- (*) 3. (a) Sound of frequency 240 Hz is normally incident on a wall with two slit-like holes separated by d . If on the other side of the wall at a distance D there is a plane for which the angular separation between the central maximum and the first minimum is 10° , determine the separation between the slits. Take $d \ll D$.
- (b) If the incident wave now has frequency 960 Hz, find the angular separation between the central maximum and third minimum.

Solution:

- (a) We set

$$d \sin \theta = \left(m + \frac{1}{2}\right) \lambda$$

where

$$\lambda = \frac{v}{f} = \frac{343}{240} = 1.429 \text{ m}$$

With $\theta = 10^\circ$ and $m = 0$ (because it is the first minimum), we obtain $d = 4.12 \text{ m}$.

- (b) Setting

$$d \sin \theta = \left(m + \frac{1}{2}\right) \lambda$$

where

$$\lambda = \frac{343}{960} = 0.357 \text{ m}$$

and $m = 2$ because it is the third minimum, we obtain $\theta = 12.52^\circ$.

- (*) 4. A 120 V rms, 60 Hz generator is placed across a circuit containing a resistor $R = 20 \Omega$ and another circuit element (either a capacitor or an inductor). The phase angle is 63° .
- (a) Identify the other element, give its reactance, and give its capacitance or inductance.
- (b) Find the average rate at which power is dissipated.

Solution:

- (a) Since the phase angle is positive and less than 90° , we know that the voltage leads the current. Thus, it is an inductor. Its reactance is given by $\tan(63^\circ) = X_L/R = X_L/20$. Thus $X_L = 39.25 \Omega$, and $L = X_L/\omega = 0.104 \text{ H}$.

- (b) The average power dissipated by the circuit is given by

$$\begin{aligned} P_{\text{avg}} &= I_{\text{rms}} V_{\text{rms}} \cos \phi \\ &= \frac{(120)^2 \cos \phi}{Z} \\ &= \frac{(120)^2 (\cos(63^\circ))}{\sqrt{(2\pi(60)(0.104))^2 + 20^2}} \\ &= 148.4 \text{ W} \end{aligned}$$

- (**) 5. An unknown inductor is in series with a $12\ \Omega$ resistor. When driven at 440 Hz, the driving voltage leads the current by 0.055 ms. Find:
- (a) The phase angle ϕ between the voltage and current.
 - (b) The reactance X_L of the inductor.
 - (c) The inductance L of the inductor.
 - (d) The impedance Z of the circuit.

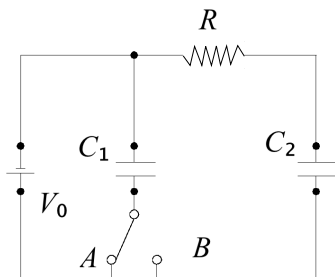
Solution:

- (a) We are told that the voltage leads the current by 0.055. The only element which has a current out of phase with the voltage would be the inductor, because a resistor's current and voltage are always perfectly in phase. Thus we know that the problem refers to the induced voltage across the inductor. Thus we set $\phi = \omega T$, where T is the amount of time by which the voltage leads the current. Thus $\phi = 2\pi f(0.055 \cdot 10^{-3}) = 2\pi(440)(0.055 \cdot 10^{-3}) = 0.152$ rad.
- (b) We know that $\tan \phi = (X_L - X_C)/R$, but since $X_C = 0$, we have $\tan(0.152) = X_L/R = X_L/12$. Thus, $X_L = 1.839\ \Omega$.
- (c) Since $X_L = \omega L = 2\pi(440)L$, we calculate $L = 0.665$ mH.
- (d) We have $Z = \sqrt{(X_L - X_C)^2 + R^2}$, but since $X_C = 0$, we have $Z = \sqrt{X_L^2 + R^2} = \sqrt{1.839^2 + 12^2} = 12.14\ \Omega$.

(**) 6. In the circuit shown in the figure, originally the switch was at A , but at $t = 0$ it is thrown to B . After a long time,

- (a) How much energy ΔE has been dissipated as heat in the resistor?
 (b) What voltages V_1 and V_2 , if any, remains on the capacitors?

Express your answers in terms of V_0 , C_1 , and C_2 .



Solution: Since the switch was at A for a long time, the capacitor C_1 will have voltage V_0 across it. The energy required to raise the voltage of C_1 to this voltage is

$$E_1 = \frac{1}{2}C_1V_0^2$$

There will also be a charge $Q_1 = C_1V_0$ on this capacitor C_1 . Both of these two quantities, energy and charge, will be conserved once the switch is flipped to position B .

- Let V_1 be the voltage on capacitor C_1 a long time after the switch is moved to position B .
- Let V_2 be the voltage on capacitor C_2 a long time after the switch is moved to position B .

We obtain one equation for the conservation of energy, and one equation for the conservation of charge. Conservation of energy:

$$\frac{1}{2}C_1V_0^2 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 + \Delta E \quad (6.1)$$

Conservation of charge:

$$C_1V_0 = C_1V_1 + C_2V_2 \quad (6.2)$$

The third equation that we need to solve this is $V_1 = V_2$. This equation is true because, after a long time, there will be no current flowing through the circuit, so the voltage across the resistor will be zero, so the voltages across the capacitors will be equal to each other.

(a) Solving the equations, the energy that has been dissipated as heat in the resistor is

$$\Delta E = \frac{1}{2} \frac{C_1C_2}{C_1 + C_2} V_0^2$$

(b)

$$V_1 = V_2 = \frac{C_1}{C_1 + C_2} V_0$$

(**) 7. An oil slick ($n_1 = 1.22$) of thickness 850 nm lies above water ($n_2 = 1.33$).

- (a) Find the colours (from 400 nm to 700 nm) that are intensified on reflection at normal incidence
 (b) Repeat for transmission

Solution: We will present two solutions here. One in terms of distance travelled by the light, and one in terms of phase shifts undergone by the light.

Solution 1

This solution will be formulated in terms of distances travelled by the light.

- (a) If the light is intensified, there is constructive interference. We need to solve the condition for constructive interference for wavelength of light in air.
- When the light is reflected from the oil's surface, there is a phase shift of π .
 - When the light is reflected from the water/oil interface there is also a phase shift of π because $n_1 < n_2$.
 - Because the light is at normal incidence, the distance that the light travels in and then back out of the oil is $2t$.

Thus we set $2t = m\lambda$, where m is an integer, and λ is the wavelength of light in the oil. The wavelengths in oil and air are related by $\lambda_{\text{air}} = n_1\lambda$. This leads to the equation

$$2t = \frac{m\lambda_{\text{air}}}{n_1}$$

Plugging in $t = 850$ nm, $n_1 = 1.22$, and trying various integer m , we find that $\lambda_{\text{air}} = 691.3, 518.15, 414.8$ nm.

- (b) This time it is destructive interference we are solving for, so we set $2t = (2m + 1)\lambda/2$, where m is an integer (and thus $2m + 1$ is an *odd* integer). This leads to the equation

$$2t = \frac{(2m + 1)\lambda_{\text{air}}}{2n_1}$$

Plugging in $t = 850$ nm, $n_1 = 1.22$, and trying various integer m , we find that $\lambda_{\text{air}} = 592.6, 460.9$ nm.

Solution 2 (Alternate)

This solution will be formulated in terms of phase shifts undergone by the light.

First, we derive an equation for phase shift upon going through a medium. Let T be the total time difference in travelling into, then back out of, an alternate medium. Then, $T = 2t/v$, where t is the thickness of the medium, and v is the velocity of light through that medium. We can rewrite $v = c/n = \lambda_{\text{air}}f/n = \lambda_{\text{air}}\omega/2\pi n$, where ω is the angular frequency of the oscillation. Then $T = 4\pi nt/\omega\lambda_{\text{air}}$. Multiplying through by ω gives $\omega T = 4\pi nt/\lambda_{\text{air}} = \delta$, where δ is the total phase shift due to extra path length.

- (a) In this part, we want the total phase shift to be $2\pi k$, for $k \in \mathbb{N}$, because we want constructive interference. When light is normally incident on the oil, it will undergo a phase shift upon reflection on the oil, and the light that is transmitted through the oil will again undergo a phase shift upon reflection at the oil-water interface. Thus the total phase shift of the light will be

$$\delta = 4\pi n_1 t / \lambda_{\text{air}} + 2\pi = 2\pi k$$

The solutions to this equation give $\lambda_{\text{air}} = 691.3, 518.5, 414.8$ nm.

- (b) In this part, we want the total phase shift to be $(2k + 1)\pi$, for $k \in \mathbb{Z}$. The total phase shift will be

$$\delta = 4\pi n_1 t / \lambda_{\text{air}} + 2\pi = (2k + 1)\pi$$

The solutions to this equation give $\lambda_{\text{air}} = 592.6, 460.9$ nm.

- (**) 8. Monochromatic light is incident on a Young's two-slit apparatus with slit separation 0.250 mm, with a screen that is 1.1 m from the slits. The third order maximum occurs at $y = 5.4$ mm.
- Determine the wavelength of the light
 - Determine the angle θ and position y on the screen of the fifth-order minimum.
 - Determine the highest-order maximum that can be observed.
 - If a flat piece of glass of index $n = 1.5$ and thickness $t = 1 \mu\text{m}$ is put in front of one of the slits, how far from the central axis will the closest maxima be?

Solution:

- (a) Using the equation

$$d \sin \theta = m\lambda$$

we find that

$$\sin \theta = \frac{5.4}{\sqrt{1100^2 + 0.25^2/4}} = 0.0049$$

Plugging that, and $m = 3$ into the equation, gives

$$\lambda = 409 \text{ nm}$$

- (b) Using

$$d \sin \theta = \lambda \left(m + \frac{1}{2} \right)$$

observe that the fifth minimum will be at $m = 4$. Thus we have

$$d \sin \theta = 4.5\lambda$$

Thus $\theta = 0.42^\circ$, and $y = 1100 \sin \theta = 8.1 \text{ mm}$

- (c) Setting $\theta = 90$ in $d \sin \theta = m\lambda$ gives $m = 611$.
- (d) The path difference due to the lens will be

$$l = t(n - 1) = 500 \text{ nm}$$

Thus, the distance equation will be

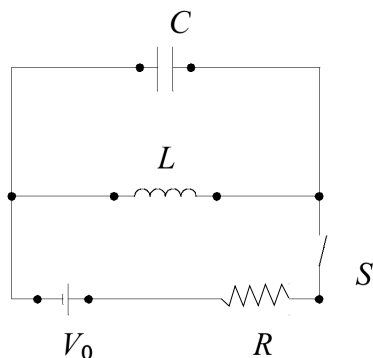
$$d \sin \theta + 500 \cdot 10^{-9} = m\lambda$$

Substituting in various values for m , we have $m = 1$ gives the minimum of

$$y_{\min} = -0.4 \text{ mm}$$

(***) 9. In the circuit shown, the switch S is initially closed, and steady current $I = V_0/R$ is flowing. At $t = 0$, S is suddenly opened.

- Find a time T after the switch is opened at which the voltage on the capacitor is maximum.
- Find the maximum voltage V_{\max} that is subsequently observed on the capacitor.
- Graph the charge on the right capacitor plate as a function of time, starting at $t = 0$. Make sure to specify any characteristic properties of the graph.



Solution: When the switch S is opened, we have only an LC oscillator. The current through the inductor at $t = 0$ will be $I_0 = V_0/R$, because that was what was going through it before the switch was opened. We can then find the current through the inductor as a function of time for $t > 0$. It will be

$$I(t) = \frac{V_0}{R} \cos\left(\frac{t}{\sqrt{LC}}\right) \quad (9.1)$$

We chose a cosine as opposed to a sine because at $t = 0$, the current is maximum.

- The charge on the capacitor is $Q = CV$. The maximum of the voltage on the capacitor will occur when the derivative is set to 0. Differentiating both sides, we see $I = C \frac{dV}{dt}$. Thus, when the derivative of V is 0, I is also 0. We can set the expression above for $I(t)$ equal to 0, and find that

$$t = \sqrt{LC} \left(\frac{\pi}{2}(2n + 1)\right)$$

for all $n \in \mathbb{Z}$. Choosing $n = 0$ gives us

$$T = \frac{\pi}{2} \sqrt{LC}$$

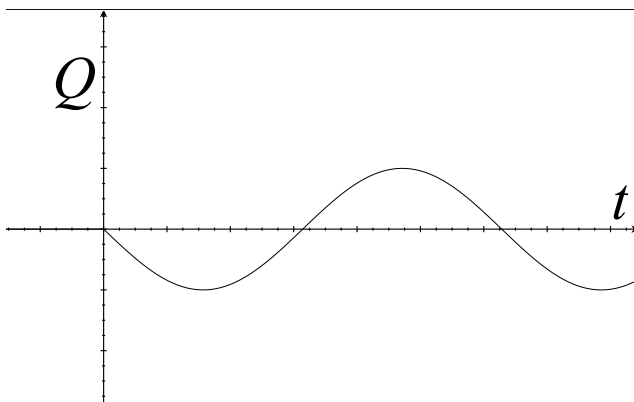
- The maximum value will be obtained by integrating the current to find Q .

$$\begin{aligned} Q &= \int_0^{\pi\sqrt{LC}/2} I(t) dt \\ &= \int_0^{\pi\sqrt{LC}/2} \frac{V_0}{R} \cos\left(\frac{t}{\sqrt{LC}}\right) dt \\ &= \frac{V_0\sqrt{LC}}{R} \\ &= CV_{\max} \end{aligned}$$

Thus we rearrange to find V_{\max} :

$$V_{\max} = \frac{V_0}{R} \sqrt{\frac{L}{C}}$$

(c) See the following figure with the charge on the right plate of the capacitor graphed vs time.



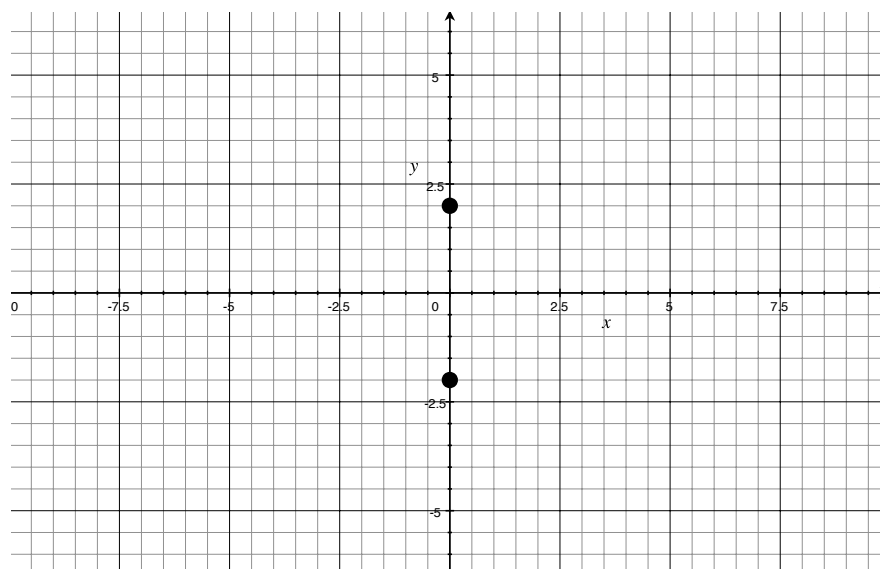
The graph is a negative sine curve, because the current starts flowing clockwise in the upper half of the circuit, which means positive charge gets deposited on the left plate of the capacitor, and thus positive charge leaves the right plate of the capacitor. The amplitude is

$$V_{\max} = \frac{V_0}{R} \sqrt{\frac{L}{C}}$$

and the period is

$$T = 2\pi\sqrt{LC}$$

- (***) 10. (a) For two in-phase point sources separated by $d = 4$ cm, if $\lambda = 18$ cm, how many curves giving maxima (constructive interference) are there? Curves giving minima (destructive interference)?
- (b) For $d = 4$ cm and $\lambda = 6$ cm, how many curves giving maxima are there? Curves giving minima?
- (c) For $d = 4$ cm and $\lambda = 1.4$ cm, how many curves giving maxima are there? Curves giving minima?

**Solution:**

If we imagine the sources being at $y = \pm 2$, then know that the lines of constructive/destructive interference will be hyperbolas opening upward and downward. Thus we can set $x = 0$ and look for y such that the following equations are satisfied: To find the locations of constructive interference, we set

$$d_1 = \sqrt{x^2 + (y - 2)^2}$$

$$d_2 = \sqrt{x^2 + (y + 2)^2}$$

and

$$d_1 - d_2 = m\lambda$$

where $m \in \mathbb{Z}$.

Setting $x = 0$, we obtain

$$|y - 2| - |y + 2| = m\lambda$$

To find the locations of destructive interference, we set

$$d_1 = \sqrt{x^2 + (y - 2)^2}$$

and

$$d_2 = \sqrt{x^2 + (y + 2)^2}$$

and

$$d_1 - d_2 = (m + 1/2)\lambda$$

where $m \in \mathbb{Z}$.

Setting $x = 0$, we obtain $|y - 2| - |y + 2| = (m + 1/2)\lambda$.

(a) Setting $\lambda = 18$, we obtain

$$|y - 2| - |y + 2| = 18m$$

There only exists a solution to this equation if $m = 0$, so we conclude that there is only one constructive interference line.

Setting $\lambda = 18$, we obtain

$$|y - 2| - |y + 2| = 18 \left(m + \frac{1}{2} \right)$$

There exist no solutions to this equation, so we conclude that there are no deconstructive interference lines.

(b) Setting $\lambda = 6$, we obtain

$$|y - 2| - |y + 2| = 6m$$

There only exists a solution to this equation if $m = 0$, so we conclude that there is only one constructive interference line.

Setting $\lambda = 6$, we obtain

$$|y - 2| - |y + 2| = 6 \left(m + \frac{1}{2} \right)$$

There exist two solutions to this equation, so we conclude that there are two deconstructive interference lines.

(c) Setting $\lambda = 1.4$, we obtain

$$|y - 2| - |y + 2| = 1.4m$$

There exist five solutions to this equation, so we conclude that there are five constructive interference lines.

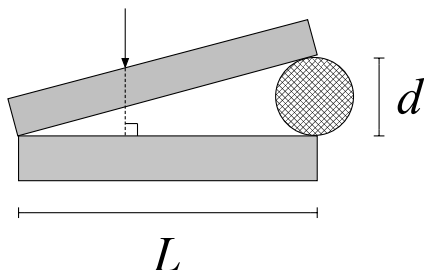
Setting $\lambda = 1.4$, we obtain

$$|y - 2| - |y + 2| = 1.4 \left(m + \frac{1}{2} \right)$$

There exist six solutions to this equation, so we conclude that there are six destructive interference lines.

- (***) 11. Consider a wedge formed by two optically flat glass plates. Suppose light is incident to the top glass plate as shown. At atmospheric pressure 1511 dark lines are observed across a thickness d for a 690.0 nm wavelength in air. When the air pressure is increased by a factor of ten, 1519 dark lines are observed.
- If the index of refraction n deviates from unity only by a term linearly proportional to the pressure, find the index of refraction of air at atmospheric pressure to six decimal places.
 - Find the wavelength of the light in vacuum.
 - Find d .

Remark. The dotted line in the figure does not denote the path of the light through the glass plate, it is there only to show the angle of incidence of light.



Solution:

- (a) The condition for destructive interference will be

$$2yn = m\lambda$$

where $m \in \mathbb{Z}$ and y is the height between the bottom plate and top plate. Note that y will vary as one moves horizontally along the wedge. Also note that $n = n(P)$, because n is a function of pressure, and $\lambda = \lambda(P)$, since λ is related to n .

Each dark line corresponds to the difference in path length of an integer plus half wavelength $\lambda(k + 1/2)$ for some integer k . Thus the last fringe, which occurs at the very end of the wedge (where $y = d$), we have the equations

$$2dn(P_{\text{atm}}) = 1511.5\lambda(P_{\text{atm}}) \quad (11.1)$$

$$2dn(10P_{\text{atm}}) = 1519.5\lambda(10P_{\text{atm}}) \quad (11.2)$$

Because the frequency of a wave of light remains constant no matter what medium it passes through, we have the equation

$$n\lambda = n'\lambda'$$

which means that

$$n(P_{\text{atm}}) \cdot \lambda(P_{\text{atm}}) = n(10P_{\text{atm}}) \cdot \lambda(10P_{\text{atm}}) \quad (11.3)$$

We are told that $n = 1 + kP$ for some constant k . Knowing that $\lambda(P_{\text{air}}) = 690 \text{ nm}$ and plugging this in to our system (9.1), (9.2), (9.3) and solving yields $k = 2.9 \cdot 10^{-9}$.

Thus, $n(P) = 1 + (2.9 \cdot 10^{-9})P$, so then $n(P_{\text{atm}}) = 1.000294$.

(b) We can use a previous equation to find the wavelength of light in a vacuum:

$$n(0) \cdot \lambda(0) = n(P_{\text{atm}}) \cdot \lambda(P_{\text{atm}})$$

Since $n(0) = 1$,

$$\lambda(0) = n(P_{\text{atm}}) \cdot \lambda(P_{\text{atm}}) = (1.000294)(690) \text{ nm} = 690.2 \text{ nm}$$

(c)

$$d = \frac{1511.5\lambda(P_{\text{atm}})}{2n(P_{\text{atm}})} = 0.52 \text{ mm}$$

Waves:

$$v = \sqrt{\frac{T}{\mu}}, \quad k = \frac{2\pi}{\lambda}, \quad P = \frac{1}{2}\mu\omega^2 A^2 v, \quad p_o = \rho\omega v s_0$$

$$v = \sqrt{\frac{\gamma RT}{M}}, \quad I = \frac{P_{av}}{4\pi r^2}, \quad \beta = 10dB \log_{10}\left(\frac{I}{I_0}\right), \quad \text{Doppler Effect } f' = f_0 \left(\frac{v \pm v_L}{v \mp v_S}\right)$$

$$\text{Beats: } \Delta f = f_2 - f_1, \quad y = A \cos(kx \mp \omega t + \phi)$$

$$\text{Interference: } k\Delta x + \Delta\phi = 2\pi n \text{ or } \pi(2n + 1), \quad n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \dots$$

$$\text{Standing Waves } f_m = \frac{mv}{2L}, \quad m = 1, 2, 3, \dots, \quad f_m = \frac{mv}{4L}, \quad m = 1, 3, 5, \dots$$

Constants:

$$k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \text{ Nm}^2/\text{C}^2, \quad \epsilon_0 = 8.84 \times 10^{-12} \text{ C}^2/\text{Nm}^2, \quad e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}, \quad c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 299,792,458 \text{ m/s}$$

Point Charge:

$$|\mathbf{F}| = \frac{k|q_1q_2|}{r^2}, \quad |\mathbf{E}| = \frac{k|q|}{r^2}, \quad V = \frac{kq}{r} + \text{Constant}$$

$$\text{Electric potential and potential energy } \Delta V = V_a - V_b = \int_a^b \mathbf{E} \cdot d\mathbf{l} = - \int_b^a \mathbf{E} \cdot d\mathbf{l}$$

$$E_x = -\frac{dV}{dx}, \quad \mathbf{E} = -\nabla V, \quad \Delta U = U_a - U_b = q(V_a - V_b)$$

Maxwell's Equations:

$$\int_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0} = 4\pi k Q_{enc} \quad \int_S \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\int_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_{enclosed}) + \epsilon_0\mu_0 \frac{d\Phi_E}{dt} \quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

Where S is a closed surface and C is a closed curve. $\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$ and $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$

Energy Density:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \text{ and } u_B = \frac{1}{2\mu_0} B^2 \text{ (energy per volume)}$$

Forces:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}, \quad \mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Capacitors:

$$q = CV, \quad U_C = \frac{1}{2} \cdot \frac{q^2}{C}, \quad \text{For parallel plate capacitor with vacuum (air): } C = \frac{\epsilon_0 A}{d}, \quad C_{\text{dielectric}} = KC_{\text{vacuum}}$$

Inductors:

$$\mathcal{E}_L = -L \frac{dI}{dt}, \quad U_L = \frac{1}{2} LI^2, \quad \text{where } L = N\Phi_B/I \text{ and } N \text{ is the number of turns.}$$

For a solenoid $B = \mu_0 n I$ where n is the number of turns per unit length.

$$\text{DC Circuits: } V_R = IR, \quad P = VI, \quad P = I^2 R$$

(For RC circuits) $q = ae^{-t/\tau} + b$, $\tau = RC$, a and b are constants

(For LR circuits) $I = ae^{-t/\tau} + b$, $\tau = L/R$, a and b are constants

$$\text{AC circuits: } X_L = \omega L, \quad X_C = 1/(\omega C), \quad V_C = X_C I, \quad V_L = X_L I$$

$$V = ZI, \quad Z = \sqrt{(X_L - X_C)^2 + R^2}, \quad P_{\text{average}} = I_{\text{rms}}^2 R, \quad I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}}$$

$$\text{If } V = V_0 \cos(\omega t), \text{ then } I = I_{\text{max}} \cos(\omega t - \phi), \text{ where } \tan \phi = \frac{X_L - X_C}{R}, \quad P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$\text{Additional Equations: } d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\mathbf{l} \times \mathbf{r}}{r^3}$$

$$\text{LRC Oscillations: } q = A_0 e^{-\frac{Rt}{2L}} \cos(\omega t + \phi), \text{ where } \omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2} \text{ and } \omega_0^2 = \frac{1}{LC}$$