Physics 158 Midterm 2 Review Package

UBC Engineering Undergraduate Society

Problems are ranked in difficulty as $(*)$ for easy, $(**)$ for medium, and $(* * *)$ for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a $(*)$ problem.

Solutions posted at: `https://ubcengineers.ca/tutoring`

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpacademic@ubcengineers.ca.

Want a warm up? These are the easier problems	Short on study time? These cover most of the material	Want a challenge? These are some tougher questions
1, 2, 3	3,4,5	7, 8,9,10

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Introduction to Electrodynamics 3 ed. / David J. Griffths
- Electricity, Magnetism, and Light / Wayne Saslow
- Exercises for the Feynman Lectures on Physics / Matthew Sands, Richard Feynman, Robert Leighton.

All solutions prepared by the EUS.

EUS Health and Wellness Study Tips

- **Eat Healthy**—Your body needs fuel to get through all of your long hours studying. You should eat a variety of food (not just a variety of ramen) and get all of your food groups in.
- **Take Breaks**—Your brain needs a chance to rest: take a fifteen minute study break every couple of hours. Staring at the same physics problem until your eyes go numb wont help you understand the material.
- **Sleep**—We have all been told we need 8 hours of sleep a night, university should not change this. Get to know how much sleep you need and set up a regular sleep schedule.

Good Luck!

(∗) 1. An AC generator produces 12 A rms at 400 V rms with power factor 1.

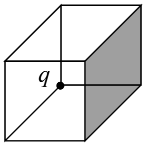
(a) Find the rms power produced by this generator.

(b) The generator voltage gets boosted by a step-up transformer to 12 kV. Find the power after the step-up transformer, assuming no losses in the transformer.

(c) The power is then transmitted to an electrical load with wires having resistance 8 Ω each way, until it reaches a step-down transformer. Determine the rms power loss in the wires.

(d) Determine the power available to the load.

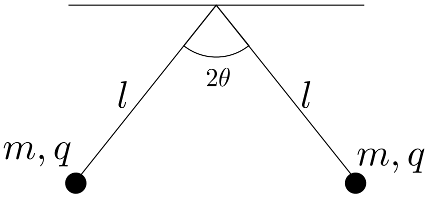
(∗) 2. Consider a charge q located at the corner of a cube. Find the electric flux through the indicated side.

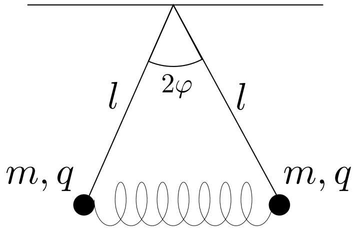


(**) 3. Suppose we have two masses with equal mass m and equal charges $+q$ on them. They are hanging from the ceiling by massless rods of length l. Let $0 < 2\theta < \pi$ be the angle between the two rods at equilibrium, when the masses are attached only to their respective rods. Suppose we want the two rods to be separated by an angle $2\varphi < 2\theta$.

(a) If an (insulating) spring of *initial* length $l_0 = l \sin\varphi$ connects the two masses, what spring constant k must the spring have in order to keep the angle between the two rods 2φ?

(b) If we introduce a uniform downward pointing electric field of strength E_0, what now is the spring constant required to keep the angle between the two rods 2φ?





(∗∗) 4. Consider an RLC circuit with $L = 2.5$ mH, $R = 4\ \Omega$, $C = 500\ \mu$F, driven by an AC voltage source of amplitude 24 V, and frequency 400 Hz.

(a) Find X_L, X_C, Z, and ϕ.

(b) Find the maximum current and maximum voltage across each of L, C, R.

(c) Find the time by which the driving voltage leads (or lags) the current.

(d) Find the maximum voltage across the combination R and C, and the time by which this voltage leads (or lags) the current.

(e) Find the maximum voltage across the combination R and L, and the time by which this voltage leads (or lags) the current.

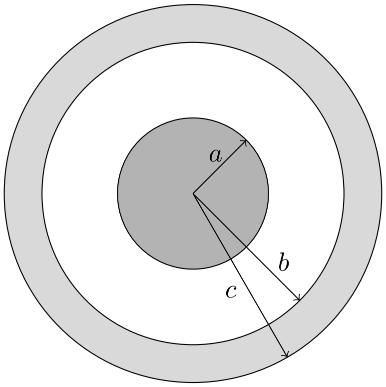
(f) Find the maximum voltage across the combination L and C, and the time by which this voltage leads (or lags) the current.

(∗∗) 5. Suppose we have a long solid insulating cylinder of radius a with volume charge density $\rho(r) = \rho_0(1-r/a)$ (in C/m^3), and a long concentric conducting shell of inner radius $b > a$ and outer radius $c > b$. There is a net linear charge density of λ (in C/m) on the conducting shell. See the cross section below.

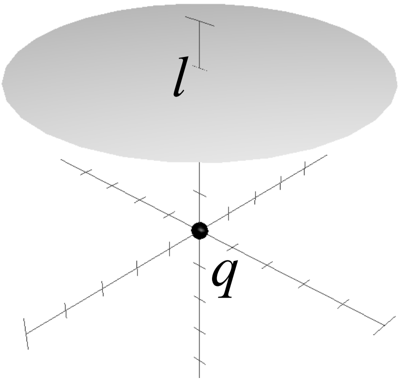
(a) Calculate the electric field as a function of r.

(b) Calculate the linear surface charge density σ_b (in C/m) on the inner surface of the conducting shell (at radius b).

(c) Calculate the linear surface charge density σ_c (in C/m) on the outer surface of the conducting shell (at radius c).



(∗∗) 6. A point charge q is at the origin. Consider a circular surface of radius a that is normal to $\mathbf{k}$, at a distance l from q. The centre of the circular surface is directly above the origin. See the figure. What is the electric flux through the surface?



(∗ ∗ ∗) 7. Imagine that the Earth were of uniform density and that a tunnel was drilled along the diameter from the North Pole to the South Pole. Assume the Earth is a perfect sphere, and let R and M be the radius and mass of the Earth, respectively. See the figure below. Also shown in the figure is a spherical Gaussian surface of radius r

(a) If an object were dropped into the tunnel, show that it will undergo simple harmonic motion.

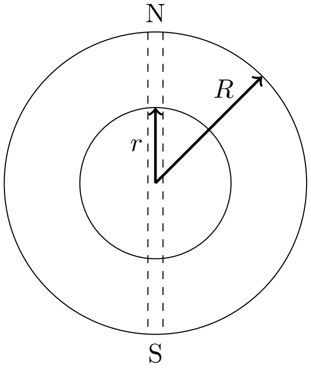
(b) Find its period P of oscillation.

(c) Show that the period P of oscillation is equal to the period of a satellite orbiting Earth just at the surface.

Hint. *Gauss's Law for gravitational fields is*

$$\oint \boldsymbol{g} \cdot d\boldsymbol{A} = -4\pi G \sum_i M_i$$

It has conceptually identical to Gauss's Law for electric fields. The analogy is $\sum Q \to \sum M$, $1/\epsilon_0 \to -4\pi G$, *and* $\boldsymbol{E} \to \boldsymbol{g}$. *Note that* $\boldsymbol{g}$ *is the gravitational field (which is analogous to the electric field* $\boldsymbol{E}$*).*



$(***)$ 8. Suppose that you have a series RLC circuit in an FM radio. You tune in on a broadcast using a variable capacitor. There are two radio stations broadcasting: Station 1 broadcasts at $\omega_1 = 6 \cdot 10^8$, and Station 2 broadcasts at $\omega_2 = 5.99 \cdot 10^8$. The inductance of the inductor is $L = 10^{-6}$ H, and both stations drive the circuit with the same max voltage.

(a) Find the value of the capacitor that you need in order to tune in to Station 1.

(b) Fix the value of C to be that found in part (a). If the mean power consumed by the circuit when listening to Station 1 (in the absence of station 2) is 100 times the mean power consumed by the circuit when listening to Station 2 (in the absence of station 1), what is the value of the resistor R?

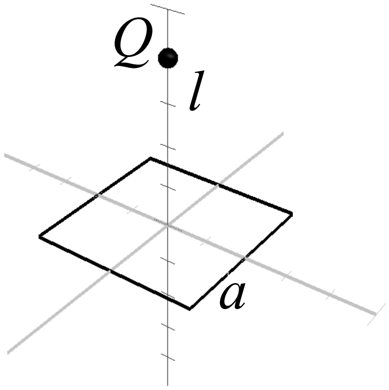
$(* * *)$ 9. A charged square insulating wire of side length a, of uniform linear charge density λ, is centred about the origin of the xy plane with sides parallel to the x and y axes. A charge Q lies a distance l above its centre. See the figure.

(a) Find the magnitude and direction of the force $\mathbf{F}$ acting on the charge Q.

(b) Find the approximate magnitude of the force $\mathbf{F}$ acting on the charge Q for $l \gg a$.

(c) Find the approximate magnitude of the force $\mathbf{F}$ acting on the charge Q for $l \ll a$.

(d) If the charge Q has a mass m, and $\lambda < 0$, and $Q > 0$, find the frequency of small oscillations of Q about the origin.



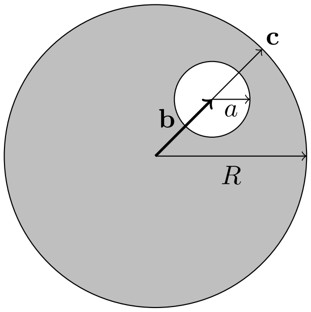
Hint: The following integral may be useful:

$$\int \frac{du}{(u^2+\alpha^2)^{3/2}} = \frac{u}{\alpha^2\sqrt{u^2+\alpha^2}}$$

($* * *$) 10. Consider an insulating sphere of radius R, centred at the origin with uniform charge density ρ. A spherical cavity of radius a is scooped out, with centre at $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, where $a + |\mathbf{b}| < R$. See the figure below.

(a) Find the magnitude and direction of the electric field at any point within the cavity.

(b) Find the magnitude and direction of the electric field at the point $\mathbf{c} = R\hat{\mathbf{b}}$, where $\hat{\mathbf{b}}$ is the unit vector in the $\mathbf{b}$ direction.



Waves:

$$v = \sqrt{\frac{T}{\mu}},\ k = \frac{2\pi}{\lambda},\ P = \tfrac{1}{2}\mu\omega^2 A^2 v,\ p_o = \rho\omega v s_0$$

$$v = \sqrt{\frac{\gamma RT}{M}}, \qquad I = \frac{P_{\text{av}}}{4\pi r^2}, \qquad \beta = 10dB\log_{10}\left(\tfrac{I}{I_0}\right), \qquad \text{Doppler Effect } f' = f_0\left(\frac{v \pm v_L}{v \mp v_S}\right)$$

$$\text{Beats: } \Delta f = f_2 - f_1, \qquad y = A\cos(kx \mp \omega t + \phi)$$

$$\text{Interference: } k\Delta x + \Delta\phi = 2\pi n \text{ or } \pi(2n+1),\ n = 0, \pm1, \pm2, \pm3, \pm4, \ldots$$

$$\text{Standing Waves } f_m = \frac{mv}{2L},\ m = 1, 2, 3, \ldots,\ f_m = \frac{mv}{4L},\ m = 1, 3, 5, \ldots$$

Constants:

$$k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9\,\text{Nm}^2/\text{C}^2, \qquad \epsilon_0 = 8.84 \times 10^{-12}\,\text{C}^2/\text{Nm}^2, \qquad e = 1.6 \times 10^{-19}\,\text{C}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\text{Tm/A}, \qquad c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 299,792,458\,\text{m/s}$$

Point Charge:

$$|\mathbf{F}| = \frac{k|q_1 q_2|}{r^2},\ |\mathbf{E}| = \frac{k|q|}{r^2},\ V = \frac{kq}{r} + \text{Constant}$$

Electric potential and potential energy $\Delta V = V_a - V_b = \int_a^b \mathbf{E}\cdot d\mathbf{l} = -\int_b^a \mathbf{E}\cdot d\mathbf{l}$

$$E_x = -\frac{dV}{dx}, \qquad \mathbf{E} = -\nabla V, \qquad \Delta U = U_a - U_b = q(V_a - V_b)$$

Maxwell's Equations:

$$\int_S \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\epsilon_0} = 4\pi k Q_{\text{enc}} \qquad \int_S \mathbf{B}\cdot d\mathbf{A} = 0$$

$$\int_C \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I_{\text{enclosed}}\right) + \epsilon_0\mu_0\frac{d\Phi_E}{dt} \qquad \int_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

Where S is a closed surface and C is a closed curve. $\Phi_E = \int \mathbf{E}\cdot d\mathbf{A}$ and $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$

Energy Density:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \text{ and } u_B = \frac{1}{2\mu_0}B^2 \text{ (energy per volume)}$$

Forces:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B},\ \mathbf{F} = I\mathbf{L}\times\mathbf{B}$$

Capacitors:

$$q = CV,\ U_C = \frac{1}{2}\cdot\frac{q^2}{C},\ \text{For parallel plate capacitor with vacuum (air): } C = \frac{\epsilon_0 A}{d},\ C_{\text{dielectric}} = KC_{\text{vacuum}}$$

Inductors:

$\mathcal{E}_L = -L\frac{dI}{dt}$, $U_L = \frac{1}{2}LI^2$, where $L = N\Phi_B/I$ and N is the number of turns.

For a solenoid $B = \mu_0 n I$ where n is the number of turns per unit length.

DC Circuits: $V_R = IR$, $P = VI$, $P = I^2R$

(For RC circuits)$q = ae^{-t/\tau} + b, \tau = RC,$ a and b are constants

(For LR circuits)$I = ae^{-t/\tau} + b, \tau = L/R,$ a and b are constants

AC circuits: $X_L = \omega L$, $X_C = 1/(\omega C)$, $V_C = X_C I$, $V_L = X_L I$

$$V = ZI,\ Z = \sqrt{(X_L - X_C)^2 + R^2},\ P_{\text{average}} = I_{\text{rms}}^2 R,\ I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}}$$

If $V = V_0\cos(\omega t)$, then $I = I_{\max}\cos(\omega t - \phi)$, where $\tan\phi = \frac{X_L - X_C}{R}, P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}\cos\phi$

Additional Equations: $d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3}$

LRC Oscillations: $q = A_0 e^{-\frac{Rt}{2L}}\cos(\omega t + \phi)$, where $\omega = \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2}$ and $\omega_0^2 = \frac{1}{LC}$