EUS Tutoring Session Review Problem Set

Mathematics 255 - Midterm 1

Ordinary Differential Equations

Note on notation: Whenever $\log(x)$ is used without a subscript to indicate the base, it is assumed to be base e in math courses. Thus in this review package, $\log(x)$ and $\ln(x)$ are used interchangeably. For inverse trigonometric functions, $\sin^{-1}(x) = \arcsin(x)$, and the other inverse trigonometric functions are similarly denoted.

The solutions to these problems will be posted on ubcengineers.ca → Services → Academic Services → Tutoring. If you believe that there is an error in an answer key, or if you have suggestions for improvement of EUS tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca.

The contents of this package include: Slope Fields, Separable Equations, First Order Linear Equations, Exact Equations, Euler's Method, Autonomous Equations, Existence and Uniqueness Theorem

1) Solve the following differential equation for $y = y(x)$. $\dfrac{dy}{dx} - \dfrac{2xy}{x^2+1} = 1$

2) Solve the following differential equation for $r = r(\theta)$. $\tan\theta \dfrac{dr}{d\theta} - r = \tan^2\theta$

3) Solve the following differential equation. $(y^2+1)dx-(x^2+1)dy=0$

4) Solve the following initial value problem. $ydy+xdx=3xy^2dx$, $y(2)=1$

5) Solve the following differential equation. $x\log xdy+\sqrt{1+y^2}dx=0$

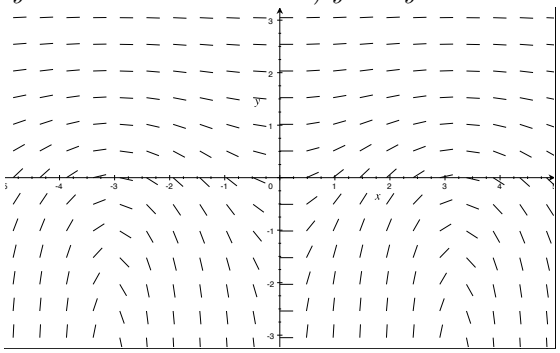
6) Solve the following differential equation. $e^{x+1}\tan y dx + \cos y dy = 0$

7) Match the following differential equations with their slope fields.

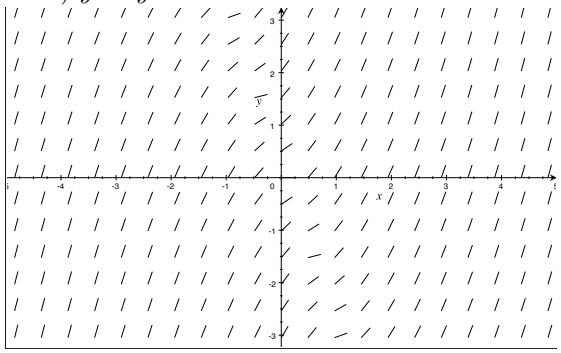
i) $y' = 4x - 2y/x$ ii) $y' = \sqrt{|3x+y|}$ iii)$y' = 1 + 2xy$

iv) $y' = e^{-y}\sin x$ v) $y' = xy$ vi) $y' = y^2 - x$

a)



b)



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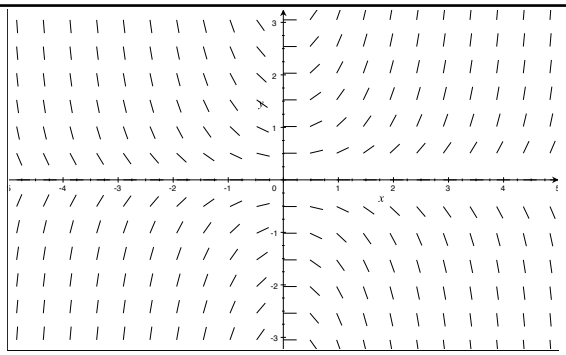

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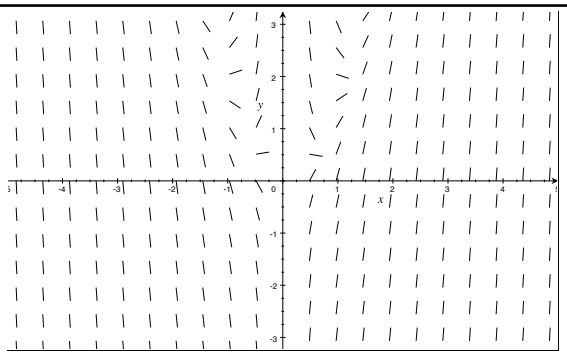

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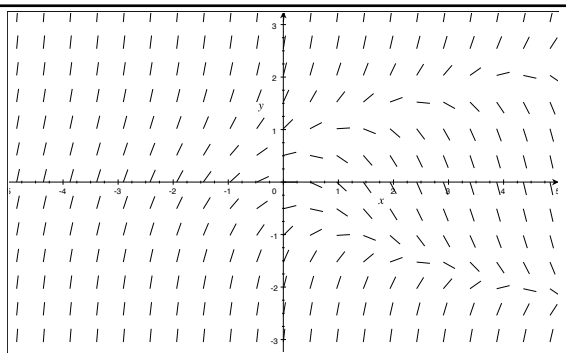
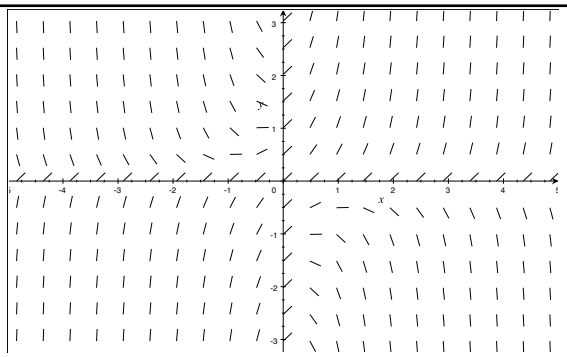


f)



8) Solve the equation. $\dfrac{dr}{d\theta} = (r + e^{-\theta}) \tan\theta$

9) Solve the equation $\tan\theta \dfrac{dr}{d\theta} - r = \tan^2\theta$

10) Solve the equation. $xy' - y = x^2 \sin x$

11) Solve the equation.$x\dfrac{dy}{dx} + 2y = \dfrac{-\sin x}{x}$

12) Create a phase diagram for the following autonomous equation. $y' = \dfrac{y-4}{y-1}$

13) Create a phase diagram for the following autonomous equation. $y' = \sqrt{y} - y^2$

14) Create a phase diagram for the following autonomous equation. $y' = y^3 + 6y^2 + 3y - 10$

15) Create a phase diagram for the following autonomous equation for $y \in [-2\pi, 2\pi]$. $y' = \sin y \log y$

16) Solve the following differential equation

$$x\sqrt{x^2+y^2}dx - \frac{x^2 y}{y - \sqrt{x^2+y^2}}dy = 0$$

17) Solve the following differential equation $(e^x \sin y + e^{-y})dx - (xe^{-y} - e^x \cos y)dy = 0$

18) Determine whether or not the initial value problem $y' = \cos(x + y)$, $y(x_0) = y_0$ has a unique solution defined on all of $\mathbb{R}$.

19) Solve the equation $2yx^3dy + (3x^2y^2 + x^3y^2 + 1)dx = 0$ (use integrating factor)

20) For the given differential equation, use Euler's Method with step size $\frac{1}{2}$ to estimate $y(2)$ if the solution passes through (1,0). $\dfrac{dy}{dx} = x - \dfrac{y^2}{4}$

21) Use Euler's Method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial value problem $y' = xy - x^2$, $y(0) = 1$.

22) Show that the function

$$y_1(x) = \begin{cases} 0, & x < 0 \\ x^3, & x \geq 0 \end{cases}$$

is a solution of the initial value problem $xy' = 3y$, $y(0) = 0$. Show that $y_2(x) \equiv 0$ is a second solution. Explain why this does not contradict the existence and uniqueness theorem.

23) Solve the following differential equation $(y^2 - 3x - 2x^2)dx + (xy - x^2)dy = 0$

24) Solve the following differential equation $x^2 + y^2 + x + xyy' = 0$

25) Solve the initial value problem: $e^x(y^3 + xy^3 + 1)dx + 3y^2(xe^x - 6)dy = 0$, $y(0) = 1$