EUS Tutoring Session Review Problem Set

Mathematics 256 - Midterm 1

Differential Equations

Note on notation: Whenever $\log(x)$ is used without a subscript to indicate the base, it is assumed to be base e in math courses. Thus in this review package, $\log(x)$ and $\ln(x)$ are used interchangeably. For inverse trigonometric functions, $\sin^{-1}(x) = \arcsin(x)$, and the other inverse trigonometric functions are similarly denoted.

The solutions to these problems will be posted on ubcengineers.ca → Services → Academic Services → Tutoring. If you believe that there is an error in an answer key, or if you have suggestions for improvement of EUS tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca.

The contents of this package include: Slope Fields, Separable Equations, First Order Linear Equations, Existence and Uniqueness Theorem, Wronskians, Second order linear constant coefficient Equations, Reduction of Order.

1) Solve the following differential equation for $y = y(x)$. $\dfrac{dy}{dx} - \dfrac{2xy}{x^2+1} = 1$

2) Solve the following differential equation for $r = r(\theta)$. $\tan\theta \dfrac{dr}{d\theta} - r = \tan^2\theta$

3) Solve the following differential equation. $(y^2+1)dx-(x^2+1)dy=0$

4) Show that the function

$$y_1(x)=\begin{cases} 0, & x<0 \\ x^3, & x\geq 0 \end{cases}$$

is a solution of the initial value problem $xy' = 3y$, $y(0) = 0$. Show that $y_2(x) \equiv 0$ is a second solution. Explain why this does not contradict the existence and uniqueness theorem.

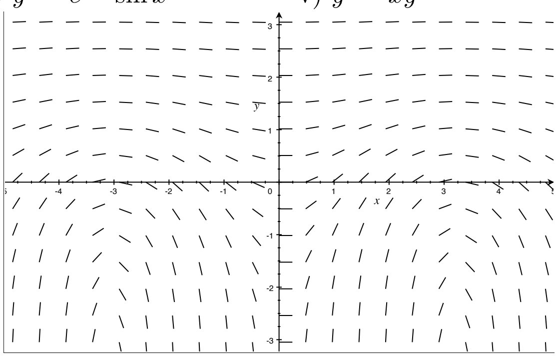
5) Determine whether or not the initial value problem $y' = \cos(x+y)$, $y(x_0) = y_0$ has a unique solution defined on all of $\mathbb{R}$.

6) Solve the following differential equation. Express the solution in terms of k. $y'' + 4ky - 12k^2y = 0$

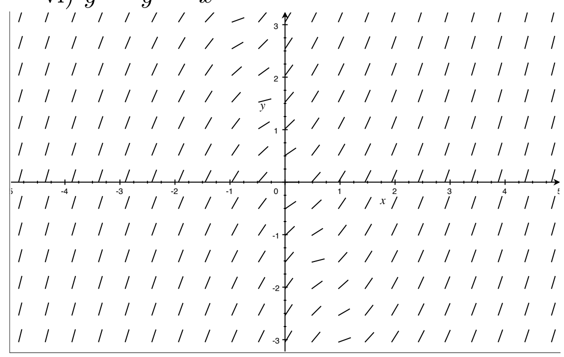
7) Match the following differential equations with their slope fields.

i) $y' = 4x - 2y/x$ ii) $y' = \sqrt{|3x + y|}$ iii)$y' = 1 + 2xy$

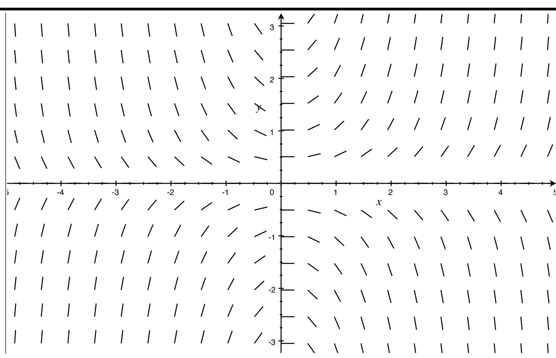
iv) $y' = e^{-y}\sin x$ v) $y' = xy$ vi) $y' = y^2 - x$



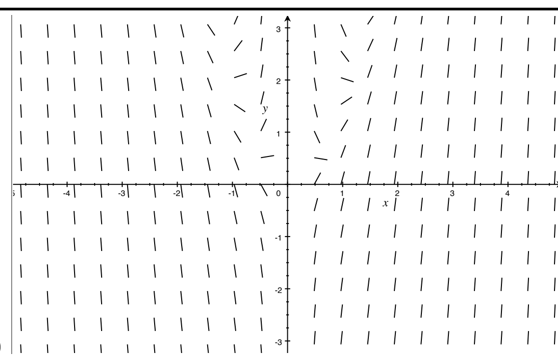
a)



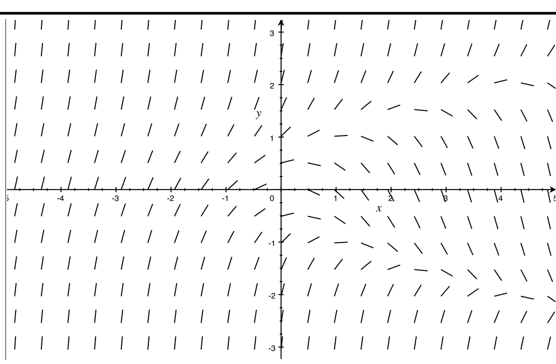
b)



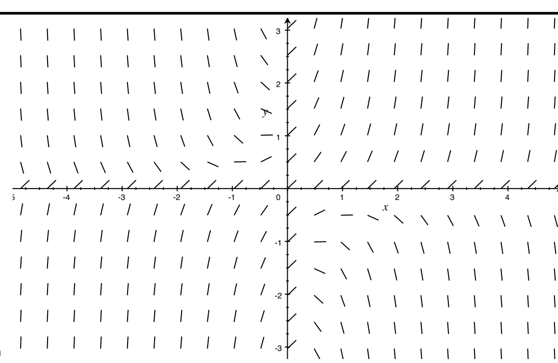
c)



d)



e)



f)

8) Solve the equation. $\dfrac{dr}{d\theta} = (r + e^{-\theta}) \tan\theta$

9) Solve the equation $\tan\theta \dfrac{dr}{d\theta} - r = \tan^2\theta$

10) Solve the equation. $xy' - y = x^2 \sin x$

11) Solve the equation. $x\frac{dy}{dx} + 2y = \frac{-\sin x}{x}$

12) Create a phase diagram for the following autonomous equation. $y' = \frac{y-4}{y-1}$

13) Create a phase diagram for the following autonomous equation. $y' = \sqrt{y} - y^2$

14) Create a phase diagram for the following autonomous equation. $y' = y^3 + 6y^2 + 3y - 10$

15) Create a phase diagram for the following autonomous equation for $y \in [-2\pi, 2\pi]$. $y' = \sin y \log y$

16) Solve the following differential equation. $x \log x dy + \sqrt{1 + y^2} dx = 0$

17) Solve the following differential equation. $e^{x+1}\tan y dx + \cos y dy = 0$

18) Assume that $y_p(x) = x^2$ is a solution of $y'' + y' - 2y = 2(1 + x - x^2)$. Find a particular solution to $y'' + y' - 2y = 6(1 + x - x^2)$.

19) Assume that $y_1 = 1+x$ is a solution of $y'' - y' + y = x$, and $y_2 = e^{2x}$ is a solution of $y'' - y' + y = 3e^{2x}$. Find a particular solution to $y'' - y' + y = x + 3e^{2x}$.

20) Find the general solution to $y''' + y'' - 6y' = 0$.

21) Find the general solution to $u'' - 2au' + a^2u = 0$

22) For the given differential equation, use Euler's Method with step size $\frac{1}{2}$ to estimate $y(2)$ if the solution passes through (1,0). $\dfrac{dy}{dx} = x - \dfrac{y^2}{4}$

23) Use Euler's Method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial value problem $y' = xy - x^2$, $y(0) = 1$.

24) If $y_1(x) = x$ is a solution to the differential equation $y'' + (x^2 - x)y' + (1 - x)y = 0$, find a second solution $y_2(x)$

25) If $y_1(x) = e^x$ is a solution to the differential equation $y'' + (x^2 - 1)y' - x^2 y = 0$, find a second solution $y_2(x)$

26) Show that each of the functions $y_1 = \sin x - \frac{1}{3}\sin 3x$ and $y_2 = \sin^3 x$ is a solution to $y'' + (\tan x - 2\cot x)y' = 0$, but that $y = c_1 y_1 + c_2 y_2$ is *not* the general solution of the differential equation.