EUS Review Problem Set

Mathematics 256 - Midterm 2

Differential Equations

Note on notation: Whenever $\log(x)$ is used without a subscript to indicate the base, it is assumed to be base e in math courses. Thus in this review package, $\log(x)$ and $\ln(x)$ are used interchangeably. For inverse trigonometric functions, $\sin^{-1}(x) = \arcsin(x)$, and the other inverse trigonometric functions are similarly denoted.

The solutions to these problems will be posted on ubcengineers.ca → Services → Academic Services → Tutoring. If you believe that there is an error in an answer key, or if you have suggestions for improvement of EUS tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca.

The contents of this package include: Second order linear equations with undetermined coefficients, variation of parameters, Laplace transforms

1) Solve the initial value problem. $y'' - 5y' + 6y = e^x(2x - 3)$, $y(0) = 1$, $y'(0) = 3$.

2) Solve the equation. $y'' + y = \sec x$

3) Solve the equation. $y'' + y' + y = x^2$

4) Solve the equation. $y'' - 3y' + 2y = \cos(e^x)$

5) Solve the equation. $y'' + 2y' + y = \dfrac{e^{-x}}{x}$

6) Solve the equation with the two solutions of the homogeneous equation given. $x^2y'' - xy' + y = x$, $y_1(x) = x$, $y_2(x) = x \log x$.

7) Solve the equation $y'' + 3y' + 2y = 8 + 6e^x + 2\sin x$

8) Solve the equation $y'' + 2y' + y = x^2 e^{-x}$

9) Solve $y'' + y = \sin(2x)\sin(x)$. The following identity may be helpful: $\sin(2x)\sin(x) = \frac{1}{2}\cos(x) - \frac{1}{2}\cos(3x)$.

10) Find the inverse Laplace transform of $F(s) = \frac{e^{-s}(s-1)}{s}$

11) Find the inverse Laplace transform of $F(s) = \frac{s+10}{s^3+2s^2+10s}$

12) Find the inverse Laplace transform of $F(s) = \dfrac{2s-1}{(4s^2+1)(9s^2+1)}$

13) Express the given function in terms of unit step functions, and then find its Laplace transform.

$$f(t) = \begin{cases} t^2 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$$

14) Express the given function in terms of unit step functions, and then find its Laplace transform.

$$f(t) = \begin{cases} 2t-1 & 0 \le t < 2 \\ t & t \ge 2 \end{cases}$$

15) Express the given function in terms of unit step functions, and then find its Laplace transform.

$$f(t) = \begin{cases} -t & 0 \le t < 2 \\ t-4 & 2 \le t < 3 \\ 1 & t \ge 3 \end{cases}$$

16) Find the inverse Laplace transforms of the following functions both in terms of step functions and in terms of piecewise defined functions. $H(s) = \frac{e^{-s}}{s^3} + \frac{e^{-2s}}{s^2}$

17) Find the Laplace transform of the following function.

$$f(t) = \int_0^t \sin(a\tau)\cos(b(t-\tau))d\tau$$

18) Find the Laplace transform of the following function.

$$g(t) = e^t \int_0^t \sin(\omega\tau)\cos(\omega(t-\tau))d\tau$$

19) Find the Laplace transform of the following function.

$$\int_0^t \tau(t-\tau)\sin(\omega\tau)\cos(\omega(t-\tau))d\tau$$

20) Find the inverse Laplace transforms of the following functions both in terms of step functions and in terms of piecewise defined functions. $H(s) = \dfrac{e^{-\pi s}(1-2s)}{s^2+4s+5}$

21) Solve the following equation using the Laplace transform. $y'' - y' - 2y = 5\sin x$, $y(0) = 1$, $y'(0) = -1$

22) Solve the initial value problem. $y'' + 9y = u(t - 2\pi)\sin t$, where $y(0) = 1$, and $y'(0) = 0$.

23) Solve the integral equation.

$$y(t) = \sin t - 2\int_0^t \cos(t - \tau)y(\tau)d\tau$$

24) Solve the integral equation.

$$y'(t) = t + \int_0^t y(\tau)\cos(t-\tau)d\tau, \qquad y(0) = 4$$

25) Evaluate the integral (Hint: Use the convolution theorem).

$$\int_0^t (t-\tau)^{13}\tau^7 d\tau$$

26) Solve the initial value problem. $y'' + 3y' + 2y = 6e^{2t} + 2\delta(t-1)$, $y(0) = 2$, $y'(0) = -6$.

27) Solve the initial value problem. $y'' + 4y = \sin t + \delta(t - \pi/2)$, $y(0) = 0$, $y'(0) = 2$

28) Solve the system.

$$\mathbf{x}' = \begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix} \mathbf{x}$$

29) Solve the system.

$$\mathbf{y}' = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix} \mathbf{y}$$

30) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -2 & -5 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$