Mathematics 100 Quiz 3 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 12 pages, including 1 cover page and 17 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (*) for easy, (**) for medium, and (***) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (*) problem.

Solutions posted at: [http://ubcengineers.ca/services/academic/tutoring/](http://ubcengineers.ca/services/academic/tutoring/)

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpaceademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

- Schuam’s Outline of Calculus 2 ed; Ayres Jr., Frank
- Calculus – Early Transcendentals 7 ed; Stewart, James
- Calculus – 3 ed; Spivak, Michael
- Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!

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1. Given \( x^2y - xy^2 + x^2 + y^2 = 0 \),

   (a) Regarding \( y \) as a function of \( x \), compute \( \frac{dy}{dx} \)

   (b) Compute a linear approximation to this graph at the point \((1, -1)\).
(***) 2. Considering $y$ as a function of $x$, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$$x^2 - xy + y^2 = 3$$
3. Prove that the two curves intersect at right angles at the origin.

(i) \( 5y - 2x + y^3 - x^2 y = 0 \)
(ii) \( 2y + 5x + x^4 - x^3 y^2 = 0 \)
4. If \( \sin y + \cos x = 1 \), compute \( y'' \) by using implicit differentiation.

5. Regarding \( y \) as a function of \( x \), compute \( y' \):

\[
x \cos y = \sin(x + y)
\]
6. Differentiate the following function. Hint: Use logarithmic differentiation.

\[
x(t) = \frac{6(1 + t^2)(t^3 - t)^2}{(4t)^{3/2}\sqrt{1 + 2t}} + \frac{\sqrt{1 + 2t}}{t + \sqrt{1 + t^2}}
\]
7. Differentiate the following function. \( y = 2\tan x \)

8. Differentiate the following function. \( y = (\tan x)^{1-x^2} \)
9. (**) Differentiate the following function. \( y = (\arcsin 3x)^{e^{-x}+1} \)

10. (**) Compute a linear approximation to \( f(x) \) at the point \( x = \pi \).

\[ f(x) = x^{\sin x} \]
11. Differentiate the following function. \( y = \arctan \left( \frac{2}{x} \right) \)

12. Differentiate the following function. \( y = x^2 \arccos \left( \frac{2}{x} \right) \)
(**) 13. Differentiate the following function. $y = \arcsin(e^x \cdot \tan x)$

(**) 14. Prove that if $f$ is an invertible, differentiable function, then

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

**Hint 1.** Recall that $f(f^{-1}(x)) = x$
(*** 15. Suppose $h$ is a function such that $h'(x) = \sin^2(\sin(x + 1))$, and $h(0) = 3$. Find

(a) $(h^{-1})'(3)$,
(b) $(\beta^{-1})'(3)$, where $\beta(x) = h(x + 1)$.)
16. Find a formula for \((f^{-1})''(x)\).

17. Suppose that \(f\) is a differentiable and invertible function, and that \(f = F'\). Let

\[ G(x) = xf^{-1}(x) - F(f^{-1}(x)). \]

Show that \(G'(x) = f^{-1}(x)\).