Mathematics 100 Quiz 4 Review Package

UBC Engineering Undergraduate Society

Attempt questions to the best of your ability. This review package consists of 3 pages, including 1 cover page and 15 questions. The questions are meant to be the level of a real examination or slightly above, in order to prepare you for the real exam. Material from lectures and from the relevant textbook sections is examinable, and the problems for this package were chosen with that in mind, as well as considerations based on past examination question difficulty and style. Problems are ranked in difficulty as (•) for easy, (••) for medium, and (•••) for difficult. Note that sometimes difficulty can be subjective, so do not be discouraged if you are stuck on a (•) problem.

Solutions posted at: http://ubcengineers.ca/services/academic/tutoring/

If you believe that there is an error in these solutions, or have any questions, comments, or suggestions regarding EUS Tutoring sessions, please e-mail us at: tutoring@ubcengineers.ca. If you are interested in helping with EUS tutoring sessions in the future or other academic events run by the EUS, please e-mail vpadademic@ubcengineers.ca.

Some of the problems in this package were not created by the EUS. Those problems originated from one of the following sources:

• Schuam’s Outline of Calculus 2 ed; Ayres Jr., Frank
• Calculus – Early Transcendentals 7 ed; Stewart, James
• Calculus – 3 ed; Spivak, Michael
• Calculus Volume 1 2 ed; Apostol, Tom

All solutions prepared by the EUS.

Good Luck!
1. Use a linear approximation to estimate the value of $29^{-1/3}$.

2. Determine the Taylor Series for $f(x) = x^6e^{2x^3}$ about $x = 0$.

3. Compute the differential of the function $y = \sqrt{3 + x^2}$.

4. Given the function $x^3 + x^2$, use a linear approximation to estimate its value at:
   (a) $x = 3.1$
   (b) $x = 3.9$

5. A culture of bacteria in a petri dish grows exponentially from an initial population of 5000 cells. Let $b(t)$ be the the number of bacteria after $t$ hours. Then $b(0) = 5000$. At $t = 10$ hours, scientists remove 4000 cells from the dish. At $t = 20$ hours, there are exactly 12000 cells in the dish. How many cells will be present in the dish at $t = 30$ hours?

6. A glass of water is left out for a long time at room temperature of 20°C. Someone puts it in the freezer which is held at a constant temperature of −5°C. At 2pm, it is 10°C, and at 3pm its just about to begin freezing. When was it put into the freezer?

7. In the year 2065, a space ship leaves Earth with the aim of finding a new planet, Paradise. The space ship initially has a population of 106 humans. The following year, the population rises to 150 humans. Assuming that the space ship has infinite capacity and the growth rate in population is according to Malthusian Model a.k.a. model of simple exponential growth. Find the population in the year 2070.

8. A body moves along a horizontal line according to the law $s = f(t) = t^3 - 9t^2 + 24t$.
   (a) When is $s$ increasing and when decreasing?
   (b) When is $v$ increasing and when decreasing?
   (c) When is the speed of the body increasing and when decreasing?
   (d) Compute the total distance travelled in the first 5 seconds of motion.

9. Two cars are moving away from each other at a speed of 60 km/hr. After half an hour one car turns left and the other turns right, both at 90° angles. What is the rate at which the distance between the cars is increasing when the cars are 50 km apart?

10. Gas is escaping from a spherical balloon at the rate of 2 cubic feet per minute. How fast is the surface area shrinking when the radius is 12 ft?

11. Two sides of a triangle are 15 and 20 m long, respectively.
   (a) How fast is the third side increasing if the angle between the given sides is $\pi/3$ and is increasing at a rate of $\pi/45$ radians per second?
   (b) How fast is the area increasing?

12. Two ships sail from the origin at the same time. One sails south at 15 km/hr; the other sails east at 25 km/hr for 1 hour and then turns north. Find the rate of rotation in rad/s of the line joining them after 3 hours.

13. Water, at the rate of 10 ft³/min, is pouring into a leaky cistern whose shape is a cone 16 feet deep and 8 inches diameter at the top. At the time the water is 12 feet deep, the water level is observed to be rising 4 inches/min. How fast is the water leaking away?
14. If the third degree Maclaurin polynomial for $g(x)$ is $g(x) \approx T_{3,g}(x) = 3 + 2x - 5x^2 + x^3/3$ (the subscript $g$ denotes it being the Maclaurin polynomial for $g(x)$), and $f(x) = (g(x))^3$, find the second degree Maclaurin polynomial for $f(x)$.

15. If $11 - 9x + 2x^2$ is the second degree Taylor polynomial to $h(x)$ at $x = 3$, and $f(x) = \frac{h(x)}{\sqrt{x + 1}}$, compute the second degree Taylor polynomial to $f(x)$ at $x = 3$. 